MATHEMATICAL MODELING OF CONFLICT AND DECISION MAKING "THE WRITER'S GUILD STRIKE 2007-2008"

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Abstract

We teach a three course core sequence in mathematical modeling directed toward decision making for our students. In this article, we will briefly describe this sequence. In the third course, we teach models of conflict stemming from Decision Theory and Game Theory. In this paper, we present the Writer's Guild Strike as an illustrative example of how Game Theory could be used to help end the strike. EXCEL is the software that we use in the course. Macros have been developed to assist in finding Nash equilibriums, testing strategic moves, finding alternative strategies, and finally finding Nash arbitration points for negotiations.

Introduction

In mathematics, you don't understand things. You just get used to them.
– Johann von Neumann

In the Department of Defense Analysis at the Naval Postgraduate School, we teach our students a three course sequence in mathematical modeling for decision making. These modeling tools are directed to give our students the ability to think analytically, quantitatively, and qualitatively in order to make "good" decisions in their jobs or prepare information for a decision maker.

Our first course covers deterministic modeling. This includes lessons that include such topics as graphs as models, dynamical systems models (using a spreadsheet), modeling fitting with least squares, and an introduction to linear programming models in both two-variable (graphical) and multi-variable with the EXCEL solver.

Our second course covers stochastic modeling. We start with a solid review of statistics and classical probability. We then cover the use of Bayes's Theorem for modeling, review of basic distributions (binomial, Poisson, uniform, exponential, and normal) and their use in modeling and reliability analysis, and we end with Monte Carlo Simulation modeling.

Our third course covers a review of expected value for use in Decision Tree Analysis. We cover decision trees, Linear Programming as a decision tool, and Game Theory. We call our third course: *Models of Conflict*.

Conflict is as ancient as humankind. Professionals spend a great deal of time studying the nature of conflict, and for that reason, our students in the Department of Defense Analysis study our modeling course sequence that allows them to use quantitative and qualitative mathematical procedures in order to help make decisions.

Models of conflict in game theory assume rational decision-makers as players in the game, trying to maximize some payoffs. In game theory, the term *rational* has a different meaning than most people think. Rational does not mean what we think is best or wise; to be rational, actors have to be able (1) to define their objectives, however foolish they appear to others, (2) to formulate sufficiently different alternative strategies, and (3) to choose a strategy that maximizes their objective. So the main question we study is: "what should / will the rational maximizing player do?"

Our approach to modeling conflict through Game Theory includes such topics as 2-person zero-sum game without communication, 2-person non-zero sum games without communications, utility theory and modeling, 2-person zero-sum game with communications, 2-person non-zero sum games with communications, n-player games with and without communications. One of our students wrote the EXCEL macros for use in the course and we thank Miroslav Feix for his work on these macros that we display in this paper.

We have many illustrative examples and case studies throughout the course to include the Cuban Missile crisis, the Gulf War, the Trump Divorce, but we would like to illustrate both game theory and the use of EXCEL with our newest example, The Writer's Guild Strike.

Writer Guild Strike

Background[1]

The 2007–2008 Writers Guild of America strike was a strike by the Writers Guild of America, East (WGAE) and the Writers Guild of America, West (WGAW) that started on November 5, 2007. The WGAE and WGAW were two labor unions representing film, television and radio writers working in the United States. Over 12,000 writers joined the strike. These entities will be referred to in the model as the Writer's Guild.

The strike was against the Alliance of Motion Picture and Television Producers (AMPTP), a trade organization representing the interests of 397 American film and television producers. The most influential of these are eight corporations: CBS Corporation, Metro-Goldwyn- Mayer, NBC Universal, News Corp/Fox, Paramount Pictures, Sony Pictures Entertainment, the Walt Disney Company, and Warner Brothers. We will refer to this group as Management.

The Writers Guild has indicated their industrial action would be a "marathon". AMPTP negotiator Nick Counter has indicated negotiations would not resume as long as strike action continues, stating, "We're not going to negotiate with a gun to our heads—that's just stupid."

The last such strike in 1988 lasted 21 weeks and 6 days, costing the American entertainment industry an estimated $500 million ($870 million in 2007 dollars).

According to a report on the January 13, 2008 edition of *NBC Nightly News*, if one takes into account everyone affected by the current strike, the strike has cost the industry $1 billion so far; this is a combination of lost wages to cast and crew members of television and film productions and payments for services provided by janitorial services, caterers, prop and costume rental companies, and the like.

The TV and movie companies stockpiled "output" so that they could possibly outlast the strike rather than work to meet the demands of the writer's and avoid the strike.

Now, let's examine this strike as an example of Game Theory.

Game Theory Approach

Let us begin by stating strategies for each side. Our two rational players will be the Writer's Guild and the Management. We develop strategies for each player.

Strategies:

- Writer's Guild: Their strategies are to strike (S) or not to strike (NS).
- Management: Salary Increase and revenue sharing (IN) or status quo (SQ).

We used the lottery method by von Neumann and Morgenstern, briefly explained later in this paper, in order to create cardinal utilities for the payoff matrix.

First, we rank order the outcomes for each side in order of preference. (These rank orderings are ordinal utilities.)

Writer's Alternatives and Rankings

- Strike- Status Quo S SQ – writer worst case (1)
- No strike – Status Quo NS SQ -- writer's next to worst case (2)
- Strike – Salary increase and revenue sharing S IN — writers next to best case (3)
- No strike – Salary increase and revenue sharing NS IN – writer best case (4)

Management's Alternatives and Rankings

- Strike - Status Quo – managements next to best case (3)
- No strike – Status Quo – management best case (4)
- Strike –Salary increase and revenue sharing—management next to worst case (2)
- No strike – Salary increase and revenue sharing – management worst case (1)

Then we provide a lottery range using the Method of von Neumann and Morgenstern [2] to enable us to create the cardinal utilities for each outcome.

- Writer's Range [*S SQ, NS IN*] from [*0,10*]
- Lottery method of von Neumann and Morgenstern provides *S* In as 6 and *NS SQ* as 4.
- Management range: [*NS IN, NS SQ*] from [*0, 10*].
- Lottery method of von Neumann and Morgenstern provides *S IN* as 2 and *S SQ* as 5.

This provides us with a payoff matrix consisting of cardinal utilities (see Figure 1). This use of cardinal utilities is important because we can then employ cardinal utilities in a Nash Arbitration scheme. We will refer to the Writers as Rose and the Management as Colin in the software.

Payoff matrix

		Management (Colin)	
		SQ	*IN*
Writer's (Rose)	*S*	(0,5)	(6,2)
	NS	(4,10)	(10,0)

Figure 1. Payoff matrix for Writer's Guild Strike.

		Management	
		SQ	*IN*
Writer's	*S*	(0,5)	(6,2)
	NS	(4,10)	(10,0)

Figure 2. Movement Diagram for Writer's Guild Strike.

John F. Nash proved that every two-person game has at least one equilibrium value either in pure or in mixed strategies. [3] The equilibriums are also called Nash Equilibriums. We can use a movement diagram to determine if a pure Nash equilibrium exists or does not exist:

Movement diagrams in non-zero sum games will be as follows: For Rose, she would maximize payoffs, so she would prefer the highest payoff at each column. Similarly for Colin, he wants to maximize his payoffs, so he would prefer the high payoff at each row. We draw an arrow to the highest payoff in that row. If all arrows point in from every direction, then that point or those points will be pure Nash Equilibrium.

We use the movement diagram (see Figure 2), to find (*4, 10*) as the Nash Equilibrium:

We notice that the movement arrows point towards (*4, 10*) as the pure Nash equilibrium. We also note that this result is not satisfying to the Writer's Guild and that they would like to have a better outcome. Both (*6,2*) and (*10,0)* within the payoff matrix provide a better outcome to the Writers.

We define the following terms used in Game Theory analysis to find an acceptable solution:

Pareto Principle: "To be acceptable as a solution of the game, an outcome should be Pareto Optimal" [4]

Pareto Optimal: The outcome where neither player can improve payoff without hurting (decreasing the payoff) the other player.

As in this case, group rationality (Pareto) is sometimes in conflict with the individual rationality (dominant). The eventual outcome depends on the players. Obtaining a Pareto optimal outcome usually requires some sort of communication and cooperation among the players.

With the assumption that the outcome should be Pareto optimal, the next question is, "What is Pareto optimal, and what is it not (Pareto inferior)?" The simplest way for this to be understood is to draw a **payoff polygon** of the game. On the chart, the X-axis depicts the payoffs of Rose and the Y-axis depicts the payoffs of Colin. By plotting the pure strategy solutions on the chart, one can see that the convex (everything inside) polygon enclosing the pure strategy solutions is then the **payoff polygon** or the **feasible region**. Therefore, the

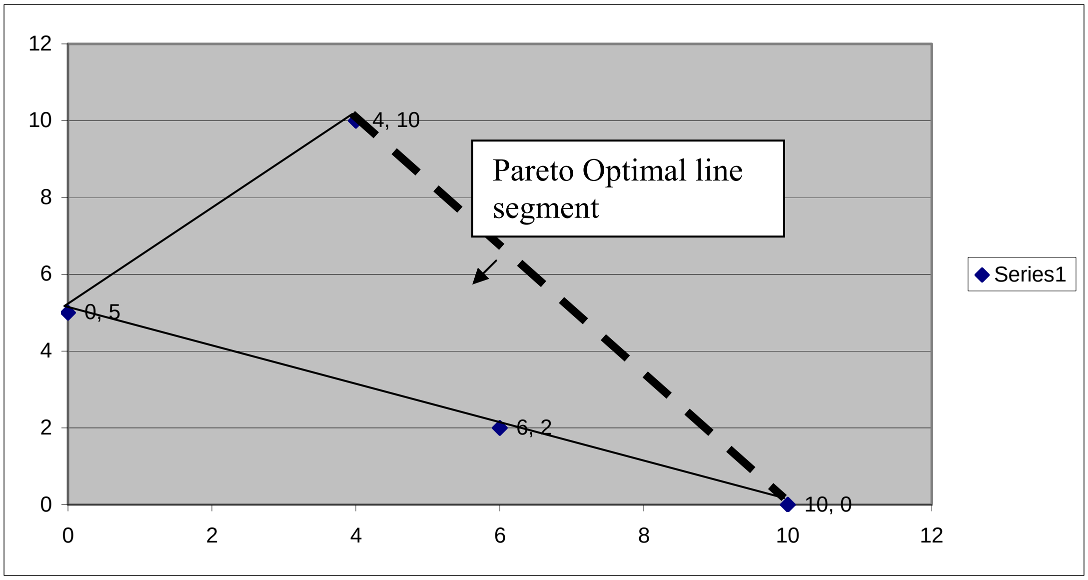

Figure 3. Payoff Polygon for Writer's Guild Strike.

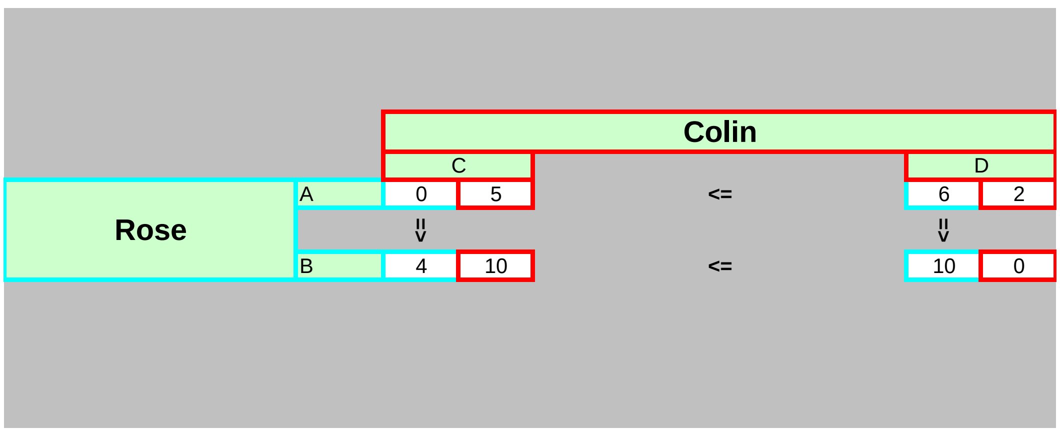

Figure 4. Payoff Matrix entry for strategic moves macro.

points inside the polygon are the possible solutions of the game.

Using Excel, we plot these coordinates from the payoff matrix to determine if any points are Pareto Optimal (see Figure 3).

The Nash equilibrium value, (4, 10), lies along the Pareto Optimal line segment. But the Writer's can do better by going on strike and forcing arbitration, which is what they did.

We can employ several options to try to secure a better outcome for the Writer's. We can first try Strategic Moves and if that fails to produce a better outcome then we can move on to Nash Arbitration. Both of these methods employ communications in the game. In strategic moves, we examine the game to see if "moving first" changes the outcome, if threatening our opponent changes the outcome, or if making promises to our opponent changes our outcome, or a combination of threats and promises in order to change the outcome. We give our student both a worksheet for Strategic Moves (see Table 1) and an EXCEL program.

In the Excel program, we start by entering the payoff matrix (see Figure 4) .

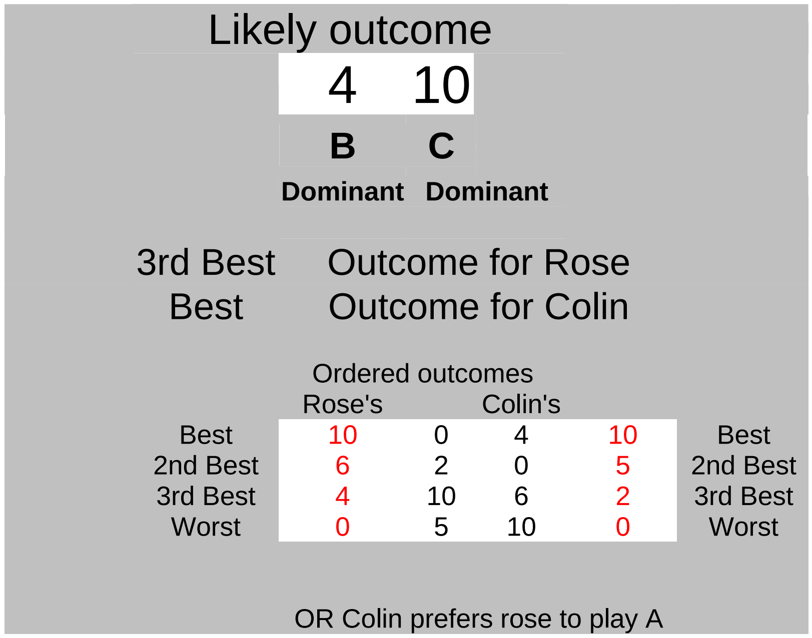

Figure 5. Likely Outcome.

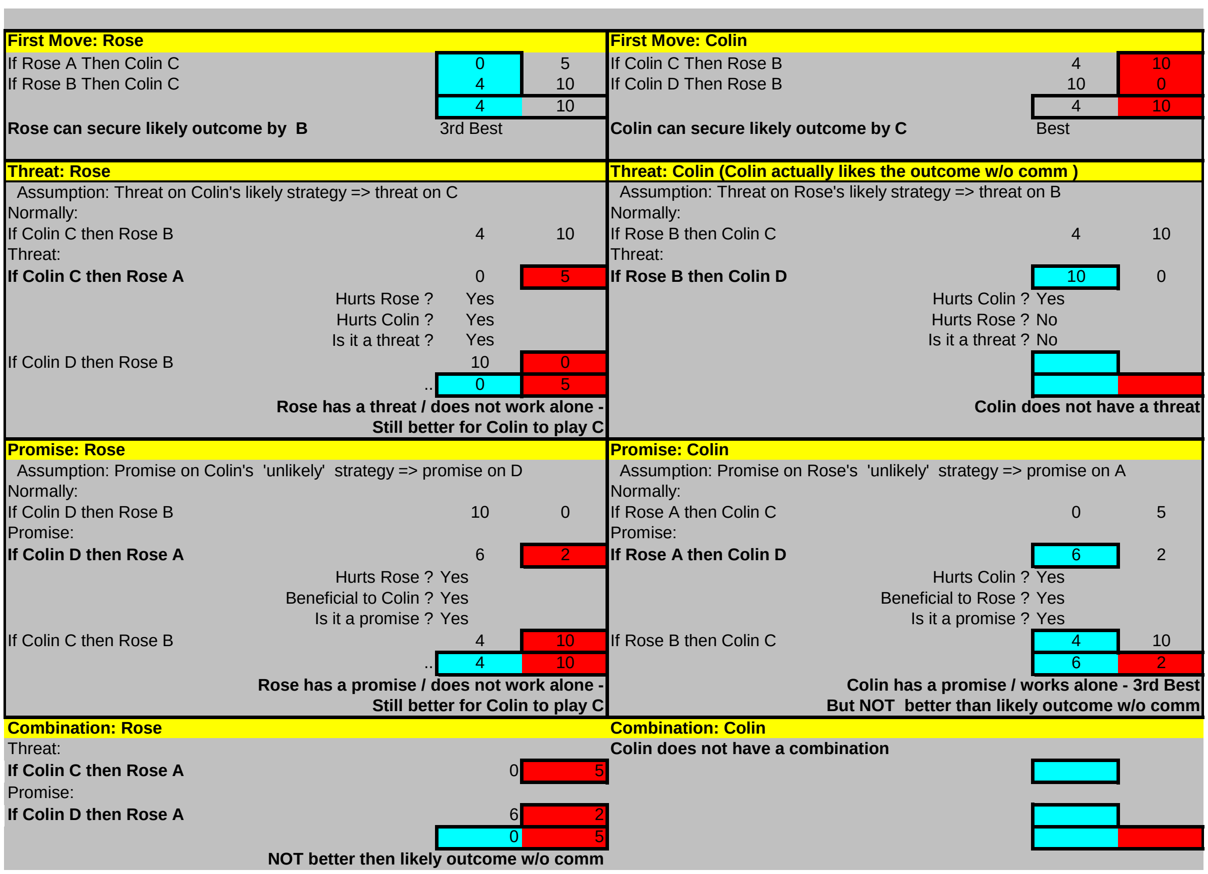

Figure 6. Strategic Moves.

The macro allows the user to find both the Nash Equilibrium (Likely Outcome) and obtain analysis of strategic moves (Solve Strategic Moves). If we put the cursor on Likely Outcomes and press enter, we obtain the Nash equilibrium result (see Figure 5).

Since we are interested in strategic moves, we enter on the Solve Strategic Moves and get the results viewed in Figure 6 (which are snapshots of the macros output).

The result of Strategic Moves is that (1) Moving first, (2) Promises, (3) Threats, nor (4) Combination of Threats and Promises did not improve the outcome from the Nash equilibrium value of (4,10). We move on to Nash Arbitration.

Nash Arbitration Scheme

Since our strategic moves did not improve our outcomes for the writer's, we consider the Nash Arbitration method. In this method, we consider "binding arbitration" where the players have a third party work out the outcomes that best meets their desires and is acceptable to all players. John Nash found that this outcome can be obtained by:

"If SQ(status quo)=(x_0,y_0), then the arbitrated solution point N is the point (x,y) in the polygon with $x \geq x_0$ and $y \geq y_0$ which maximizes the product $(x- x_0)*(y- y_0)$." [6]

Table 1: ANALYSIS FOR STRATEGIC MOVES [5]

- **Simultaneous Without Communication**
  - Dominant Strategies? Existence of Nash Equilibrium?
  - **Conclusion**: The likely outcome without communication **(__, __)**
- **With Communication (Strategic Moves) from Rose's Perspective**
- **FIRST MOVES**

  Should Rose move first:

  If Rose does A, then Colin does __ , implies outcome (__ , __)

  If Rose does B, then Colin does __ , implies outcome (__ , __)

  So Rose would choose outcome (__ ,__)

  Should Rose **force** Colin to move first:

  If Colin does C, then Rose does __ , implies (__ , __)

  If Colin does D, then Rose does __ , implies (__ , __)

  So Colin would choose (__ , __)

  Conclusions: Rose moving first would result in outcome (__ , __)

  Forcing Colin to move first would result in outcome (__ , __)
- **THREATS**: **Example**: Suppose Rose wants Colin to play D

  If Colin does C and Rose does the opposite of what she logically should do (in order to hurt herself) , then Rose does, __ with outcome (__, __)

  Does it also hurt Colin? If so, it is a threat and eliminates outcome (__, __)

  With the threat working , Colin chooses __ and the outcome is (__ , __)

  Does the threat work alone? (Does she in fact get Colin D?)
- **PROMISES: Example**: Suppose Rose wants Colin to play D

  If Colin does D and Rose hurts herself, she does Rose __with outcome (__, __)

  Does it help Colin? If so, it is a promise and eliminates (__, __)

  With the promise working, the outcome is (__ , __)

  Does the promise work alone? (Does she in fact get Colin D?)
- **COMBINATION THREAT AND PROMISE**

  Threat eliminates (__ . __) **AND** the Promise eliminates (__ . __)

  Logical outcome is (__ . __)
- **Summary of Strategic Moves available to Rose (and to Colin)**

		Colin			
		C		<=	D	
Rose	A	0	5	<=	6	2
		⇓			⇓	
	B	4	10	<=	10	0

Rose - Prudential Strategy - Solution

Rose's Game
Rose Maiximizing
Colin Minimizing the opponent's payoff

Zero-Sum Game with Rose's Payoffs (blue)

			Colin		
		C		D	
	A	0	<=	6	
Rose		⇓		⇓	
	B	4	<=	10	Dominant
		Dominant			

Solution in the Pure Strategy: Yes — **Value of the Game: 4** 4

Solution in the Mixed Strategy: No — **Rose's Security level**

Rose should play (Rose - Prudential Strategy)

A	**0.00%**	**% of the time and**	0
B	**100.00%**	**% of the time**	1

When Colin plays

C	**100.00%**	**% of the time and**	1
D	**0.00%**	**% of the time**	0

Colin - Prudential Strategy - Solution

Colin's Game
Colin Maiximizing
Rose Minimizing the opponent's payoff

Zero-Sum Game with Colin's Payoffs (red)

			Colin	
		C		D
	A	5	<=	2
Rose		⇑		⇓
	B	10	<=	0
		Dominant		

Solution in the Pure Strategy: Yes — **Value of the Game: 5** 5

Solution in the Mixed Strategy: No — **Colin's Security level**

Colin should play (Colin - Prudential Strategy)

C	**100.00%**	**% of the time and**	1
D	**0.00%**	**% of the time**	0

When Rose plays

A	**100.00%**	**% of the time and**	1
B	**0.00%**	**% of the time**	0

Figure 7. Prudential Strategies.

Status quo point in the definition is the likely outcome of the game when the negotiation fails. An arbitrated solution should be better for both players than the status quo; this is incorporated in the definition by $x \geq x_0$ and $y \geq y_0$. Status quo is the minimum the players can get. Everything above is improvement of their gain. The solution has to maximize their joint utility. The objective function – $(x- x_0)*(y- y_0)$, maximizes these 'above security level' utilities. In other words, it has to maximize the area of the rectangle.

The status quo point is the security levels of each side. We find these values using prudential strategies. Again, the software can assist us in finding these values as (4,5). We start by entering the outcomes into the payoff matrix and requesting the Prudential Strategies. A snapshot is found as Figure 7 illustrating the value (4,5) as the security levels for our two players. The security level is an important component in the Nash arbitration scheme.

We can input these security level values, (*4*, 5), into the program for finding the Nash arbitration values.

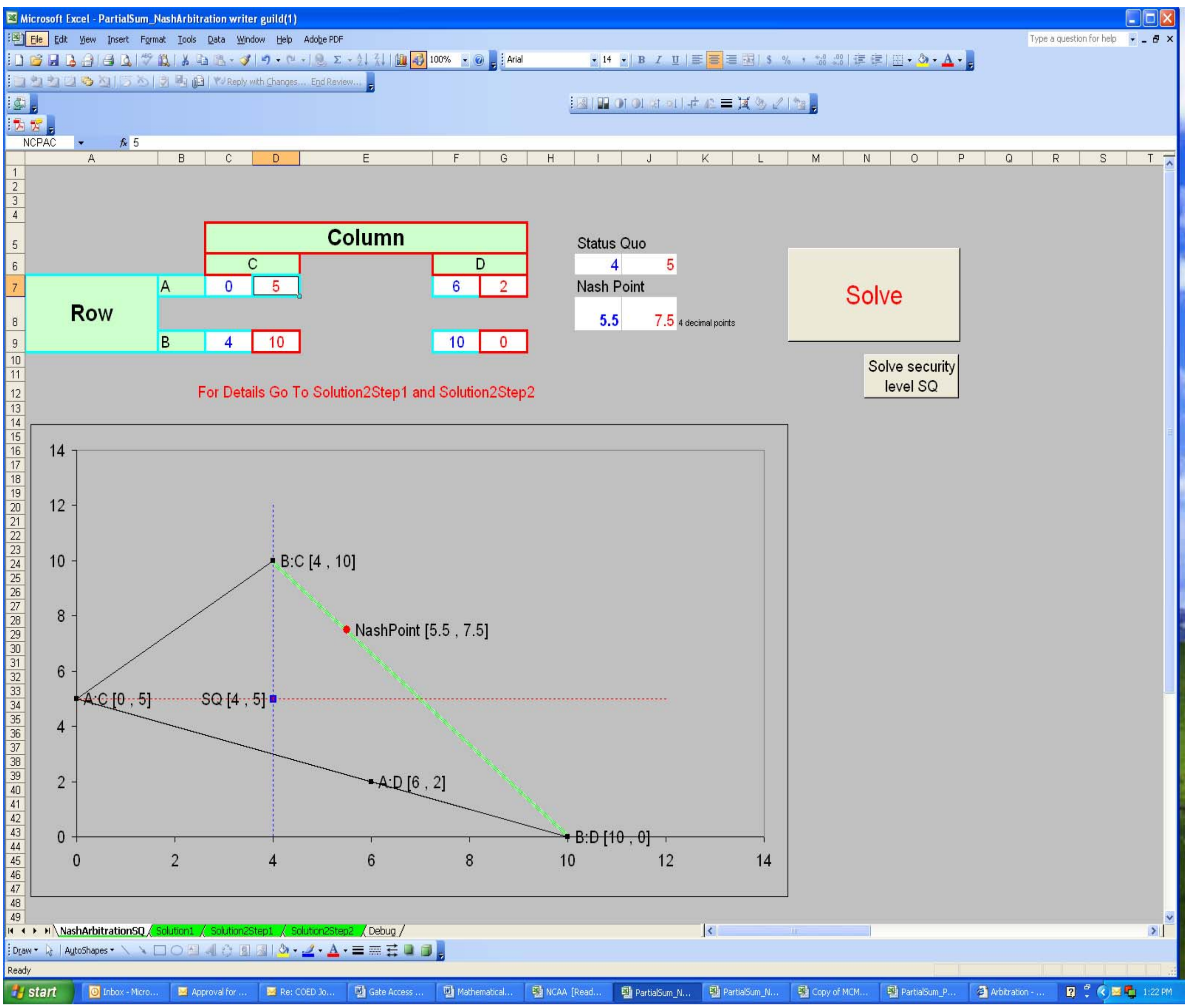

Figure 8. Nash Arbitration result for the Writer's Guild.

Results

One of the strong elements of the Nash arbitration is that by using the security levels as the basis of the arbitration that our players can do no worse than those values (4, 5). For the Writer's Guild that is a value of 4 that is equal to their value under the Nash equilibrium. That means that worst case their outcome is at least the equilibrium value. The Excel programs enabled our students to find the Nash Arbitrated solution value of *(5.5, 7.5).* The Writer's Guild should be able to improve their outcome from 4 to 5.5 through an arbitrated solution. This is shown in Figure 8.

How should the negotiators achieve this result?

We find that the negotiators should consider the following strategy: Writer's should not strike and the Management should offer status quo at 75% and the salary increasing and revenue sharing at 25%.

The Nash Arbitrated solution was found to be *(5.5, 7.5).* The Writer's Guild was able to improve their outcome from 4 to 5.5 through an arbitrated solution. This is a substantial increase for the Writer's and would effectively satisfy both players in the game.

Conclusions

The analysis is made more accessible by the use of the Excel macros, especially since all the original outcomes in the pay-off matrix are subjectively obtained from the players and could change. The programs provide the student with the ability to obtain a quick, efficient, and accurate analysis of the situation. The macros are easy to use and to change the values in the pay-off matrix.

We find that building models for current events as well as for historic events is interesting to our students. The student's always provide additional 'insights' that lead to interesting discussions about 'what ifs' and the macros enable us to explore these insights and what-if analysis.

References

1. Wikipedia, http://en.wikipedia.org/wiki/2007_Writers_Guild_of_America_strike, *The Writer's Guild Strike*, 2007.

2. John Von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior*, 60th anniversary ed. Princeton, N.J.; Woodstock: Princeton University Press, 2004.

3. John F. Nash, "Equilibrium Points in n-Person Games," *Proceedings of the National Academy of Sciences of the United States of America* 36, (1). January 15, 1950.

4, Philip D.Straffin, *Game Theory and Strategy*, New Mathematical Library, Vol. 36, Washington: Mathematical Association of America, 1993.

5. Frank R. Giordano, *Course Notes for Modeling under Conflict*, Naval Postgraduate School, 2007.

6. John F. Nash, "The Bargaining Problem," *Econometrica,* 18, (2) April, 1950.

Biographical Information

Dr. William P. Fox received his BS degree from the United States Military Academy at West Point, New York, his M.S. at the Naval Postgraduate School, and his Ph.D. at Clemson University. He has taught at USMA, at Francis Marion University, and is currently at the Naval Postgraduate School He has authored two textbooks on mathematical modeling. He has over one hundred technical and educational articles, presentations, and workshop presentations. He serves as the associate contest director for COMAP's Collegiate Mathematical Contest in Modeling (MCM) and serves as contest director for the High School Mathematical Contest in Modeling (HiMCM). His interests include applied mathematics, optimization (linear and nonlinear), mathematical modeling, statistical models for medical research, and computer simulations.