QUICK-RETURN MECHANISM REVISITED

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Abstract

In this paper, the teaching and learning experiences of the author with two summer interns at one of the educational institutions in India and other students at Kettering University in Flint, MI are presented. The students in India were the senior mechanical engineering students from two different engineering colleges who spent nearly two months at the Indian institute where the author spent a 3-month sabbatical as a visiting faculty member. Although these two students took the "Theory of Machines" course at their college, a complete understanding of kinematic and dynamic analyses of mechanisms such as a quickreturn linkage seemed to be not fully realized by them, due to the limitation of laboratory and CAE facilities at their parent colleges. The other mechanical engineering students at Kettering University took a basic course on Dynamics of Rigid Bodies, but they too had no hands on laboratory or CAE tools used for this course. Therefore, they took the Design and Analysis of Mechanical Systems and Assemblies course which is offered as a mezzanine level directed study course. Previously recorded lecture material was sent to distance learning students, while the recorded lectures helped the on campus students as supplementary material. The students modeled the linkages using the motion simulation application that is commonly available in any CAE tool such as Catia, UG-NX, NX I-DEAS, or SolidWorks. Other math tools such as MatLab Simulink, Excel, MapleSim, etc., are also used to to perform calculations and to draw plots of the various characteristics. All student participants were taught the basics of loop-closure equations pertaining to the kinematic and dynamic analysis of planar mechanisms with an example of quickreturn, which is the topic of the paper presented here. For the quick return mechanism, plots such as variation of quick return ratio as a function of the critical link lengths, and kinematic and dynamic characteristics of the linkage have been Ankita Sahu and Theodore Paul Dani Summer Interns Indian Institute of Technology – Gandhinagar Ahmedabad, Gujarat – 383824

studied and validated by CAE tools. Studies related to partial balancing of the system are also under way, mostly using a CAE tool. Finally, the students in India used the available laboratory experimental apparatus to verify some of the theoretical calculations. The performance metric is a final report that included the learning outcomes and recommendations for further work.

Introduction and Literature Review

The Course Learning Objectives (CLOs) of the course are:

- 1. *Apply* the integration of the fundamental concepts of rigid body kinematics in relative motion, solid mechanics and computer aided engineering through computational and design tools.
- 2. *Apply* fundamental mechanics principles to the kinematic, dynamic and fatigue stress analyses of components of planar mechanisms, subsystems and systems.
- 3. *Use* state-of-the-art CAE software tools to formulate, conceptualize, design, analyze, and synthesize open-ended problems pertaining to mechanical systems.
- 4. *Develop* strategies to improve the product and process design based on the results obtained.

In tune with the above CLOs, students in the course were taught using a combination of theory using graphical and analytical methods and CAE tools such as I-DEAS or NX 7.5. However, covering in depth theory using analytical kinematics is found to be challenging due to time constraints. On the other hand, when using the conventional graphical methods, although limited to analysis of a mechanism in the instantaneous reference frame, students seem to realize their

ease of use. In order to expand the students understanding of mechanisms, it is important to explore the same system at multiple points in time, delivering an understanding of the cycle the mechanism would go through during operation. It is common for students and researchers to explore the use of software to design and analyze mechanism cycles using CAE tools such as Catia, Unigraphics-NX, HyperWorks, NX I-DEAS or SolidWorks [1-5]. Also mathematical tools such as MapleSim [6] and Matlab[7] and written programs such as C⁺⁺, Fortran and T-K Solver [8-10], etc are also used. However, the use of these programs does not require the student to have a deeper understanding of the methods being used in the analysis. In the case of software such as C^{++} , Matlab, and Maplesim the student does not have a visual representation of how the model behaves while in motion. Conversely, to use solid modeling and simulation software such as Solidworks and UG-NX the student is not required to have a full understanding of the methods being used in the analysis. In order to allow students to analyze an example model while still understanding the methods involved, an analysis program for a Whitworth quick-return mechanism was created in Microsoft excel. The same model was modeled in CAE software and motion was simulated to create a reference for verification of the excel model. The model was further cross referenced to a previously published work on the use of a C^{++} program that provided a solution to the example in question[11]. The example used is the Whitworth quick-return system, an uncommonly explored linkage because it has not been used in high speed applications due to fundamental difficulties with unbalance and vibrations. The Microsoft Excel program generated in this report is eventually to be expanded upon with the interest of exploring methods to balance the system, reduce vibrations in and expand on the applications of this mechanism. A physical model was subsequently used by the students in India to acquire data and verify the authenticity of the equations and models. All students were recommended to use the online Statics modules to review their knowledge of rigid body equilibrium, center of gravity and moment of inertia concepts.

Analysis

The model analyzed in this paper was replicated from a previous paper that explored the use of C^{++} programing to fully model a Whitworth Quick return mechanism published by Matt Campbell and Stephan Nestinger [11]. In the interests of cross referential verification, the model simulated in this paper is the same as the model in the previously published paper.



Figure 1. Vector representation of the Whitworth Quick Return Mechanism. [11]

The equations for Kinematic analysis that are presented below were originally presented by Campbell and Nestinger [11].

Position Analysis:

The displacement analysis can be formulated using the following equations:

$$r_1 + r_2 = r_3$$
 (1a)

$$r_3 + r_8 + r_5 = r_6 + r_7 \tag{1b}$$

Using complex numbers, Equations 1 and 2 become

$$r_3 r_1 e^{i\theta_1} + r_2 e^{i\theta_2} = r_3 e^{i\theta_3}$$
 (2a)

$$e^{i\theta_3} + r_8 e^{i\theta_8} + r_5 e^{i\theta_5} = r_6 e^{i\theta_6} + r_7 e^{i\theta_7}$$
 (2b)

Here, link lengths r_1 , r_2 , r_4 , r_5 , r_7 and angular positions θ_1 , θ_6 , θ_7 are constants.

Now, equation1 becomes

$$r_3 e^{i\theta_4} = r_1 e^{i\theta_1} + r_2 e^{i\theta_2} \tag{3a}$$

Since $r_4 = r_3 + r_8$, equation 4 can be written as,

$$r_4 e^{i\theta_4} + r_5 e^{i\theta_5} - r_6 e^{i\theta_6} = r_7 e^{i\theta_7}$$
 (3b)

Using Euler's equation,

$$e^{i\theta} = \cos\theta + i\sin\theta,$$

$$r_3 \cos\theta_4 = r_1 \cos\theta_1 + r_2 \cos\theta_2$$
(4a)

$$r_3 \sin \theta_4 = r_1 \sin \theta_1 + r_2 \sin \theta_2 \tag{4b}$$

Squaring equations 4, and adding them together, we get

$$r_{3} = \sqrt{(r_{1}\cos\theta_{1} + r_{2}\cos\theta_{2})^{2} + (r_{1}\sin\theta_{1} + r_{2}\sin\theta_{2})^{2}}$$
(5)

Dividing equation 4b by Equation 4a and simplifying gives

$$\theta_4 = \tan^{-1} \left(\frac{r_1 \sin \theta_1 + r_2 \sin \theta_2}{r_1 \cos \theta_1 + r_2 \cos \theta_2} \right) \tag{6}$$

Now, equation (3b) can be written as

$$r_6 e^{i\theta_6} - r_5 e^{i\theta_5} = r_4 e^{i\theta_4} - r_7 e^{i\theta_7}$$
(7)

Since, the right hand side of equation 7 is constant, let us consider

$$re^{i\theta} = r_4 e^{i\theta_4} - r_7 e^{i\theta_7}$$

This is used in further calculations. Now, equation 7 becomes

$$r_6 \cos \theta_6 - r_5 \cos \theta_5 = r \cos \theta \tag{8a}$$

 $r_6 \sin \theta_6 - r_5 \sin \theta_5 = r \sin \theta \tag{8b}$

Solving equations 8 for r_6 gives

$$r_{6a} = \frac{r\cos\theta + r_5\cos\theta_{5a}}{\cos\theta_6} \tag{9a}$$

$$r_{6b} = \frac{r\sin\theta + r_5\sin\theta_{5a}}{\sin\theta_6} \tag{9b}$$

Here, equation (9a) is used when $cosr_6 > 0$ and equation 9b is used when $cos \theta_6 = 0$. Substituting Equation (9a) into equation (8b) gives

$$\sin(\theta_5 - \theta_6) = \frac{r\cos\theta\sin\theta_6 - r\sin\theta\cos\theta_6}{r_5} \qquad (10)$$

Solving for θ_5 , we find

$$\theta_{5a} = \theta_6 + \sin^{-1} \left(\frac{r \cos \theta \sin \theta_6 - r \sin \theta \cos \theta_6}{r_5} \right)$$
(11a)
$$\theta_{5b} = \theta_6 + \pi - \sin^{-1} \left(\frac{r \sin \theta \cos \theta_6 - r \cos \theta \sin \theta_6}{r_5} \right)$$

Knowing all of the angular positions and the length of r_6 , we can find the position of the output slider, link 6, using

$$P_6 = r_4 + r_5 \tag{12}$$

(11b)

Velocity Analysis:

The velocity analysis can be formulated by taking the time derivative of equation (2) which is as follows:

$$\dot{r}_{3}e^{i\theta_{4}} + r_{3}i\omega_{4}e^{i\theta_{4}} = \dot{r}_{1}e^{i\theta_{1}} + r_{1}i\omega_{1}e^{i\theta_{1}} + r_{2}i\omega_{2}e^{i\theta_{2}}$$
(13a)

$$\dot{r_6}e^{i\theta_6} + r_6i\omega_6 e^{i\theta_6} = \dot{r_4}e^{i\theta_4} + r_4i\omega_4 e^{i\theta_4} + \dot{r_5}e^{i\theta_5} + r_5i\omega_5 e^{i\theta_5} - \dot{r_7}e^{i\theta_7} - r_7i\omega_7 e^{i\theta_7}$$
(13b)

Here, $\dot{r_1} = \dot{r_2} = \dot{r_4} = \dot{r_5} = 0$ (as these links are assumed to be rigid members), $\omega_1 = 0$ (since link 1 is rigid), $\omega_6 = \omega_7 = 0$ and $\theta_6=0$ as links 6 and 7 are assumed to be non-rotating imaginary members, and $\dot{r_7} = 0$ because the output slider 6 is assumed to remain on the ground at all times. Taking these considerations into account, we have

$$\dot{r_3}e^{i\theta_4} + r_3i\omega_4e^{i\theta_4} = r_2i\omega_2e^{i\theta_2}$$
(14a)

$$\dot{r_6} = r_4 i \omega_4 e^{i\theta_4} + r_5 i \omega_5 e^{i\theta_5} \tag{14b}$$

Solving for ω_4 , we have

$$\omega_4 = \frac{r_2 \omega_2 \cos \theta_2 \cos \theta_4 + r_2 \omega_2 \sin \theta_2 \sin \theta_4}{r_3}$$
(15)

Substituting equation (15) into either the real or imaginary equation of equation (14a)

Equations (14) can be written as,

$$\dot{r_6} = -r_4\omega_4\sin\theta_4 - r_5\omega_5\sin\theta_5 \tag{16a}$$

$$0 = r_4 \omega_4 \cos \theta_4 + r_5 \omega_5 \cos \theta_5 \tag{16b}$$

From equation (16b),

$$\omega_5 = -\frac{r_4 \omega_4 \cos \theta_4}{r_5 \cos \theta_5} \tag{17}$$

Thus, the velocity of output slider can be found by using

$$V_6 = \dot{r_6} + \dot{r_7} \tag{18}$$

Since, the vertical component of velocity, $V_{6y}=0$, $V_{6x}=\dot{r}_{6}$

Acceleration Analysis:

The acceleration analysis has been formulated by taking the first time derivative of equations (14). Breaking up equation (14a) into its real and imaginary parts, we have

$$\dot{r}_{3} = \sqrt{(r_{2}\omega_{2}\cos\theta_{2} - r_{3}\omega_{4}\cos\theta_{4})^{2} + (r_{3}\omega_{4}\sin\theta_{4} - r_{2}\omega_{2}\sin\theta_{2})^{2}}$$
(19)

The angular acceleration of link 4 was found as

$$\alpha_4 = \frac{r_2}{r_3} \{ -\omega_2^2 \cos(\theta_2 - \theta_4) + \alpha_2 \sin(\theta_2 - \theta_4) \} - 2 \frac{r_3}{r_3} \omega_4$$
(20)

The linear acceleration of the output slider is given as

 $\begin{aligned} \ddot{r_6} &= -r_4 \alpha_4 \sin \theta_4 - r_4 \omega_4^2 \cos \theta_4 - r_5 \alpha_5 \sin \theta_5 - r_5 \omega_5^2 \cos \theta_5 & (21) \\ 0 &= r_4 \alpha_4 \cos \theta_4 - r_4 \omega_4^2 \sin \theta_4 + r_5 \alpha_5 \cos \theta_5 - r_5 \omega_5^2 \sin \theta_5 & (22) \end{aligned}$

From equation (22), we get,

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$$\alpha_5 = \frac{r_4(\omega_4^2 \sin \theta_4 - \alpha_4 \cos \theta_4) + r_5 \omega_5^2 \sin \theta_5}{r_5 \cos \theta_5}$$
(23)

Substituting α_5 in equation (21), we get acceleration of the output slider, i.e.

$$a_6 = \ddot{r_6} + \ddot{r_7} \tag{24}$$

Since the vertical component of acceleration, $a_{6y} = 0$, $a_{6x} = \ddot{r}_6$

Dynamic force analysis equations can also be programmed in Excel following the same process. This, together with balancing is in progress.

Following sections present the results of the analytical study and discuss the learning outcomes of the students.

Results and Discussion

Analytical Model

The first analysis completed in the Excel programs was the variance of the quick return ratio, or QRR, of the inverted slider crank loop as a function of its component links which were individually varied. The result visually demonstrated to the students what had been shown in the equations. As shown in Figure 2 b, for each case the system has an asymptotic relationship to its bounding conditions, i.e., the link lengths can never be equal or else the QRR approaches infinity as the link lengths approach each other.

The result of varying the QRR by changing the link lengths was further explored by examining the components in the first loop. In Figures 3 and 4 the baseline is the behavior of the model that was replicated from Campbell and Nestinger [11], and by varying the link lengths the behavior of the respective links within the first loop are altered. Figure 3 examines the changes in the slider position relative the ground pivot point. In all cases the slider oscillates between the difference and the sum of the crank and ground link lengths. Varying the ground link will vary the average value, while varying the crank length will vary the amplitude of the oscillation. Figure 4 examines the effects of QRR on the angle of the output arm link 4. As the arm continues to oscillate about the vertical position, the average value for all cases remains 90 degrees, or $\frac{\pi}{2}$ radians. Varying the QRR has the same effect at the extremes for variance of either link length. As the length difference approaches zero, the slope of theta 4 approaches infinity at the bottom dead center position. Conversely, as either the ground length (L1, same as r1 in Figure 1) approaches infinity or

the crank length (L2, same as r2) approaches zero, the amplitude of the oscillation approaches zero.

The first point of comparison between the excel program, the simulation results from the CAE simulation, in this case Unigraphics NX7.5, and the results published by Campbell and Nestinger [11] was the velocity analysis of the output slider link 6. As shown in Figures 5 through 7, the results across all three are the same.



Figure 2. QRR as ground link and crank link lengths are varied.



Figure 3. Slider Position Relative to Ground Pivot as QRR approaches extremes.



Figure 4. Loop 1 Output Arm Angle as QRR approaches extremes.



Figure 5. Excel Result for Output Slider Velocity versus Time.



Figure 6. Unigraphics NX7.5 Result for Output Slider Velocity versus Time.



Figure 7. Output Slider Velocity versus Time as published by Campbell and Nestinger[11].

In the course of verification between Campbells findings and the output from the Unigraphics NX7.5, and the result of the excel program, the students noticed that error and discontinuity were introduced close to the so called "dead center positions" if simply following the equations presented by Campbell and Nestinger [11]. As these discontinuities were corrected, the students gained an improved understanding of the time variance in the system. Similar processes were completed in calculating the accelerations of each link, and subsequently the forces on each link and at each joint. This will be presented in the final draft.

Physical Model and Data Acquisition System:

In order to better understand the kinematics of the mechanism, the summer intern students in India conducted experiments at the Indian Institute of Technology (IIT) – Gandhinagar to analyze the behavior of the system and the various conditions associated with it. The tests done in the laboratory on a physical model with data acquisition and sensors helped the students in better visualization of the actual system and the associated data acquisition and measurements. This also helped the students to better understand the various types of sensors used for the measurement. A breadboard model at Kettering University is also available that can be modified and assembled to demonstrate the motion of the quick-return and several other planar mechanisms. The quick-return model available in the KDM (Kinematics and Dynamics of Machines) Laboratory at the IIIT-GN is shown in Figure 8.



Figure 8. Physical model of a Whitworth Quick Return system Used.

The major components of this apparatus are the motor, Quick return links (crank, connecting rod, slider, fixed link), sensors (Accelerometer, Tachometer), and the data acquisition system.

Motor:

The motor is a constant speed type that derives power from the electrical source. The motor has a provision to be driven at different constant speeds. The motor is coupled with the rotating disc.

Quick Return Links:

The various links are assembled and are driven by the motor at various speeds. In this apparatus the crank radius can change for 5 different crank positions. For these various crank radii the quick return ratio is calculated.

Accelerometer Sensors:

The accelerometer sensor that is used in this apparatus is of capacitive type. These capacitors operate in a bridge circuit, along with two fixed capacitors, and alter the peak voltage generated by an oscillator when the structure undergoes acceleration. Detection circuits capture the peak voltage, which is then fed to a summing amplifier that processes the final output signal.

Tachometer:

The tachometer is a device used to measure the speed of a rotating object. The tachometer used here is of inductive type. The variation of the air gap induces a pulse which is counted by a counter and the rpm is counted.

Data Acquisition System:

The data acquisition system takes the analog output from the various sensors and converts them into digital values by means of an Analog to Digital converter. This digital analogous value is fed into the processing unit which does the required processing and gives the output to the display unit. The computer uses software called KDM (Kinematics and Dynamics Of Machines). The needed values and their characteristic curves are plotted by the software and the output is recorded.

The charts, as shown in Figure 9, were the outputs when the crank length was 12.7 mm and the system was being driven at the crank rotating speed of 70 rpm.



Figure 9. Outputs of physical data acquisition lab setup.

KDM –Software: - The DATA ACQUISITION system takes the analog input from the sensors and converts it to digital signals and processes it further using the KDM software to generate the various characteristic curves. The parameters that are plotted have not been given any units since they are just the voltage equivalent of the output. The maximum and minimum values of the displacement, velocity and acceleration can be observed from the graph. These plots do not show results for full cycle of operation.

Various experiments have been performed by changing the input speed and the crank radius to plot the linear velocity and acceleration of the slider which changes with respect to variation in crank speed and crank radius. The students also learned that the graphs, although look smooth, have some noise from signals and also from vibrations caused by the moving links that are not rigid in real life.

As mentioned, the physical model allowed students to vary the crank length and measure the resultant change on the output slider while observing the changing behavior of the system. In the absence of a physical model, students were

able to conduct similar experimentation for the inverted slider crank loop of the complete mechanism by using the Catia CAE tool. Another example of such a CAE model using UG-NX 7.5 is shown in Figure 10.



Figure 10. Whitworth Quick Return modeled in Unigraphics NX7.5 for simulation.

The dimensions used for this model are the same as those presented in the literature.

These CAE models allow the student to construct similar experiments and generate several plots while retaining the visual demonstration of the apparatus. Further studies on balancing of this linkage are in progress. These results will be presented in a separate paper at another conference.

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Conclusion

Based on the example work presented in this paper, the students have demonstrated an increased depth of understanding of planar mechanism theory via the creation and verification of their graphical and analytical models using math and CAE tools such as Excel and NX while also retaining a solid grasp of the physical system via either the data acquisition apparatus or the virtual CAE model. In doing so, they have explored and defined the various limiting link conditions (dead center position) of the Whitworth Quick Return system and the ramifications of the said conditions. The variance of the ground link length and/or the crank length and thus the QRR alters the system behavior and increases the understanding of the applicability of the linkage for applications involving a quick return cycle. Further studies to partially balance the linkage will be undertaken by the future students of this class. The learning outcomes written by the students indicate that they learned the theory well when complimented with use of a simulation tool such as a CAE or a MATH tool. Further, they appreciated the use of a real experimental apparatus, which enabled them to understand better the measurement system and their limitations based on a comparison of the theoretical and experimental results.

Although in this paper only a quick return mechanism is presented, other planar mechanisms using higher pairs (cams and gears) are also studied using both graphical and analytical methods, as well as, analysis using a simulation tool such as UG-NX. Integration of all the learning tools enables the students to learn better. The assessment tools used were the homework, laboratory reports and a comprehensive examination covering all aspects of planar mechanisms.

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At the time of carrying out this work, both Theodore Paul Dani and Ankita Sahu were senior undergraduate students of mechanical engineering from different colleges in India. Both of them spent two months as Summer 2012 Interns at I.I.T., Gandhinagar, and worked under the guidance of Dr. Echempati. Currently, both of them work for different engineering industries in India. Ms. Ankita Sahu works as a Software Engineer at Capgemini India Pvt Ltd., Plant no. 5, Godrej & Boyce Mfg Co Ltd., Pirojsha Nagar, LBS Marg, Vikhroli(West) Mumbai - 400 079. Mr. Theodore Paul Dani works as a Graduate Engineer Trainee in the R & D part of Product Development, Renault-Nissan Technology And Business Centre India, Ascendas IT Park, Chengalpet, Kancheepuram District, Tamil Nadu -603002.