USING EXCEL TO ENHANCE UNDERSTANDING OF VIBRATION ANALYSIS BY FINITE ELEMENT TECHNIQUE

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Introduction

Vibration analysis is an essential subject for structural engineers, and the most efficient and accurate method for handling complicated vibration analysis is using the finite element technique. There are several well developed commercially available finite element software packages. However, in order for one to be able to properly apply the software, a fair understanding of the development techniques of the software is necessary. Consequently, from an educational point of view, it is necessary to familiarize the students with the logic behind the development of the software products before using them.

In order to truly understand the logic behind the development process of a finite element software product, a student must initially set up the finite element equations and then use a computer to solve the equations. After the initial understanding of the technique, a commercial finite element software can be used to analyze similar but larger structures.

A typical engineering student may not have the necessary computer programming skills to efficiently set up the equations and to solve them by computerized numerical analysis techniques. EXCEL provides a tool for setting up and solving the finite element equations for simple structures, without requiring an extensive computer programming effort.

The theme of this article is using EXCEL for setting up free undamped vibration equations using the finite element technique and solving them by computerized numerical techniques. The article presents the application of the suggested teaching technique by presenting an example of analyzing a simple truss using EXCEL and then extending the simple example to set up the structure for a section of an aircraft wing and, then, solving the extended structure by ANSYS finite element software.

Presentation of free undamped vibration equations

The free undamped vibration equation for a one degree of freedom system is shown in equation (1).[1]

$$m.(d^2d/dt^2) + s.d = 0 \qquad (1)$$

For a multi degree of freedom system, the free undamped vibration equation becomes as shown in equation (2).[1] In equation (2) and throughout this article, the symbols "[" and "]" are used for defining matrices.

$$[M][d^2D/dt^2] + [S][D] = [0] \qquad (2)$$

By using the principle of virtual work, the mass [M] and stiffness [S] matrices for any vibrating system can be obtained. [M] and [S] are energy equivalent mass and stiffness matrices, and [D] is the displacement matrix containing the displacements for the chosen degrees of freedom for the specified locations of the vibrating system. The symbol *d* indicates integration, and t represents time. The transformation from the local coordinate system to the global coordinate system, and vice versa, is accomplished by using a rotation matrix.[1]

Equation (2) has a known solution that is shown in equation (3).[1]

$$D_i = \Phi_i \sin(\omega_i t + \alpha_i) \quad (i=1, 2, \ldots\ldots\ldots, n) \quad (3)$$

In equation (3), n is the number of degrees of freedom of the system, Φ_i is a vector of nodal amplitudes (the mode shape) for the ith mode of vibration, ω_i is the vibration frequency for the ith mode, and α_i is the phase angle for the ith mode.

It can be shown that expansion of equation (4) results in a polynomial of degree n, and the solution of the resulting polynomial results in n values for ω_i. The mode shapes for each vector can then be obtained by back substituting the values of ω_i into equation (2).

$$|S - \omega_i^2 M| = 0 \qquad (4)$$

Example of developing free undamped vibration equations for a truss system and solving the equations

Trusses are made of beams. In order to be able to assemble mass and stiffness matrices for trusses, the mass and stiffness matrices for individual beam elements must be defined.

The mass matrix for a beam element is defined by equation (5).[1]

$$[M] = \rho A L/420 \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4LL & 13L & -3LL \\ 54 & 13L & 156 & -22L \\ -13L & -3LL & -22L & 4LL \end{bmatrix} \qquad (5)$$

The stiffness matrix for a beam element is defined by equation (6).[1]

$$[S] = (2EI/L^3) \begin{bmatrix} 6 & 3L & -6 & 3L \\ 3L & 2LL & -3L & LL \\ -6 & -3L & 6 & -3L \\ 3L & LL & -3L & 2LL \end{bmatrix} \qquad (6)$$

In equations (5) and (6), ρ is the density of the beam material, A is the cross sectional area of the beam, L is the length of the beam element, and E is the modulus of elasticity of the beam material.

It must be noted that the matrices shown in equations (5) and (6) apply to the local axis of the beam element. If local axes for a beam element are not parallel to global axes for the whole structure, rotation of axes transformation must be used for each nodal displacement, acceleration, stiffness, and mass, before they can be used in the format shown in equation (2).[1]

A free undamped vibration equation for a truss system in the form of equation (2) can be obtained by breaking the system up into a number of beam elements and then obtaining the mass and stiffness equations for each beam element in the global coordinate system and combining the matrices where there are common nodes.

Consider the truss shown in Figure 1. Node 1 is fixed in vertical direction. This kind of restraint can be accomplished by placing the node on a roller that permits horizontal movement and prevents vertical movement. Node 2 is fixed in both horizontal and vertical directions.

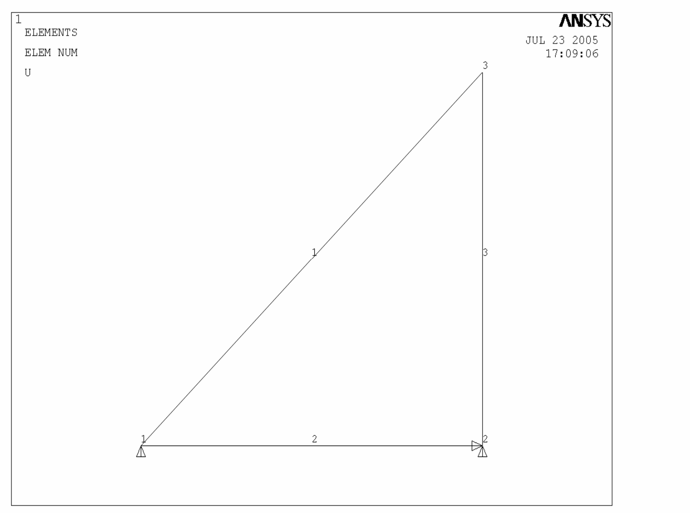

Figure 1: Illustration of a three element truss.

The stiffness matrices shown in equations (7), (8) and (9) can be derived for elements 1, 2 and 3 of Figure 1 in the global coordinate system by using equation (6), and then conducting a coordinate transformation for element 1.

$$S_1 = (EA/L)\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{bmatrix} 0.36 & 0.48 & -0.36 & -0.48 \\ 0.48 & 0.64 & -0.48 & -0.64 \\ -0.36 & -0.48 & 0.36 & 0.48 \\ -0.48 & -0.64 & 0.48 & 0.64 \end{bmatrix} \end{array} \tag{7}$$

$$S_2 = (EA/L)\begin{array}{c} \begin{matrix} 1 & 2 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{8}$$

$$S_3 = (EA/L)\begin{array}{c} \begin{matrix} 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \tag{9}$$

The numbers shown above each of the matrices in equations (7), (8) and (9) correspond to a degree of freedom. Numbers 1 and 2 correspond to degrees of freedom in horizontal and vertical directions for node 1, numbers 3 and 4 correspond to degrees of freedom in horizontal and vertical directions for node 2, and numbers 5 and 6 correspond to the degrees of freedom in horizontal and vertical directions for node 3.

The 3 matrices shown in equations (7), (8) and (9) can be combined to find a global matrix for the entire structure shown in Figure 1. Equation (10) is the global matrix for the complete truss system of Figure 1.

$$S_{total} = (EA/L)\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1.36 & 0.48 & -0.36 & -0.48 & -1 & 0 \\ 0.48 & 0.64 & -0.48 & -0.64 & 0 & 0 \\ -0.36 & -0.48 & 0.36 & 0.48 & 0 & 0 \\ -0.48 & -0.64 & 0.48 & 1.64 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array} \tag{10}$$

The total mass matrix can be obtained by combining the mass matrices for the individual elements. Each individual mass matrix can be obtained based on equation (5) and a subsequent matrix rotation if applicable. Equation (11) is the global mass matrix for the complete truss system.

$$Mtotal = (\rho a l/6)\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 2.72 & 0 & 1 & 0 & 0.36 & 0 \\ 0 & 2.72 & 0 & 1 & 0 & 0.36 \\ 1 & 0 & 3.28 & 0 & 0.64 & 0 \\ 0 & 1 & 0 & 3.28 & 0 & 0.64 \\ 0.36 & 0 & 0.64 & 0 & 2 & 0 \\ 0 & 0.36 & 0 & 0.64 & 0 & 2 \end{bmatrix} \end{array} \tag{11}$$

The matrices shown in equations (10) and (11) can be put into equation (4), and the natural frequencies (ω_i's) of the system can be determined. All the steps described for the matrix formation and the subsequent determination of the natural frequencies for a small system, such as the one shown in

Figure 1, can be programmed into an EXCEL spread sheet without any software development abilities. The natural frequencies obtained for the first three modes of vibration for the system of Figure 1 by using EXCEL is shown in Table 1.

Table 1: First three natural frequencies
Of the system shown in figure 1.

Mode number	Frequency(Hz)
1	1205700
2	3000100
3	4428900

Using an EXCEL spread sheet for setting up the vibration equations and solving the equations provides invaluable insight into the development philosophy of the finite element technique.

Application of truss analysis techniques to analyzing the vibration characteristics of aircraft wings

The primary structure in an aircraft wing consists of a number of connected truss structures. One of the considerations in an aircraft wing analysis is the determination of the vibration characteristics of the wing. Figure 2 is an illustration of the internal structure for an aircraft wing section.[3]

While using an EXCEL spread sheet is possible for setting up and solving a structure such as the one shown in Figure 1, use of EXCEL for solving a structure like the one shown in Figure 2 is not an efficient use of one’s time. Consequently, The ANSYS finite element model was used for determining the natural frequencies and the mode shapes for the structure shown in Figure 2.

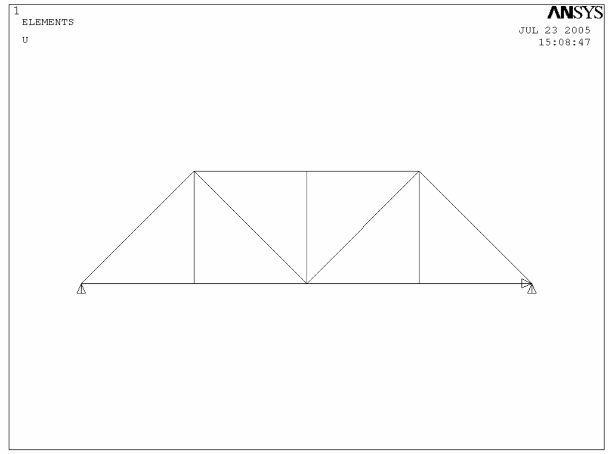

Figure 2: Illustration of the internal structure for an aircraft wing section.

Table 2 contains the first three natural frequencies for the structure shown in figure 2.

Table 2: First three natural frequencies
Of the system shown in figure 2.

Mode number	Frequency(Hz)
1	376740
2	644800
3	1079600

Figures 3, 4 and 5 contain the mode shapes for modes 1, 2 and 3 of the structure shown in Figure 2. The mode shapes shown in Figures 3 through 5 are obtained from ANSYS.

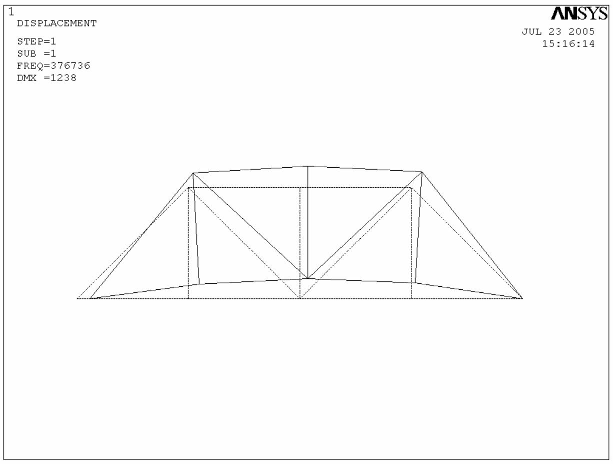

Figure 3: Mode shape for mode 1 of structure of Figure 2.

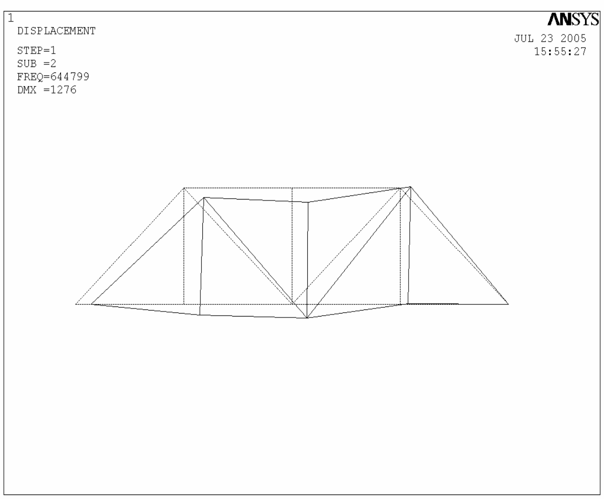

Figure 4: Mode shape for mode 2 of structure of Figure 2.

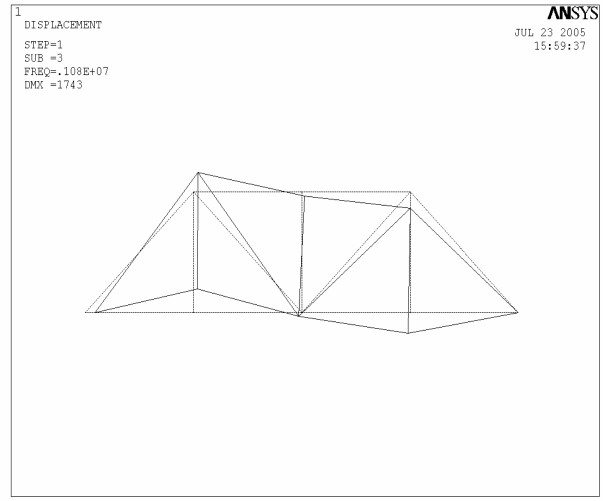

Figure 5: Mode shape for mode 3 of structure of Figure 2.

Justification for using EXCEL as an initial teaching tool

The required matrix manipulations described in this article, and solving of the polynomial equation for determination of the natural frequencies of the vibrating system, and determination of the mode shapes require extensive computational efforts for even small systems. Performing the required mathematical manipulations without the use of computers is tedious, and do not contribute to gaining insight into the application of the finite element technique for solving vibration problems.

Using the EXCEL spreadsheet software to perform the repetitive mathematical manipulations for small systems will free the students to try different scenarios quickly without having to resort to any software development or direct use of a commercial finite element software. This will enable the students to efficiently set up the finite element equations by manual techniques giving them invaluable insight into the techniques and rational used for developing the finite element software. Freeing the students to concentrate on learning the vibration analysis and finite element techniques in place of learning computer programming or performing tedious math will contribute to a more efficient and productive use of the students' time.

Summary and closing remarks

In this article a technique used by the author in a graduate level engineering technology course for teaching the use of finite element technique in structural analysis is presented. The article is presenting the use of the EXCEL spread sheet for setting up and solving the finite element equations as related to undamped free vibrating systems, and then using ANSYS for analyzing a bigger version of the problem solved by EXCEL.

It must be emphasized that the technique described in this article also applies to more complex vibration scenarios such as damped and forced vibrating systems. The technical manuals for most commercially available finite element software describe a number of different approaches for solving various problems. The author has found that it is impossible to evaluate the applicability of the various techniques to a given situation without setting up and solving the problems manually. The technique suggested in this article makes it possible for an individual without a great deal of software development ability to quickly set up small problems and solve them outside of the finite element package.

References

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Biographical Information

Dr. Hagigat is teaching undergraduate and graduate engineering technology courses at The University of Toledo. Dr. Hagigat has an extensive industrial background, and he is continuously emphasizing the practical applications of engineering material covered in a typical engineering technology course.