NONLIN: SOFTWARE FOR STRUCTURAL DYNAMICS AND EARTHQUAKE ENGINEERING EDUCATION

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Abstract

NONLIN is a computer program that has been specifically designed to enhance learning in graduate university classes and continuing education seminars that cover subjects related to structural dynamics and earthquake engineering. The program is highly interactive, graphically intensive, and user friendly. This paper describes the basic features of *NONLIN*, and provides several suggestions for use in an academic environment.

Programming Philosophy

NONLIN was programmed using *Microsoft* Visual Basic version 6.0. All of the problem input is provided interactively. The program has a variety of error and data completion checking procedures. For example, the program will not allow the user to attempt an analysis unless all of the data has been provided, nor will it allow access to previously computed results if system parameters have changed.

NONLIN is graphically intensive in both the input and the output. The various graphs displayed by the program may be viewed on the screen, printed, or saved in tab-delimited Microsoft *Excel* format for future processing.

The basic principle in programming *NONLIN* is "minimum click design". Whenever possible, an intuitively labeled button is provided to access a feature instead of requiring the user to navigate through a hierarchy of menus. For example, a ground motion acceleration record may brought into a model with only two mouse clicks (one to select the ground motion tool, one to select the earthquake). Two additional clicks provide a series of response spectra for the ground motion (one to select the tool, one to plot the spectra). Once a response history has been computed and plotted the Fourier amplitude spectrum of the response may be computed by a single mouse click. Fourier spectra may be obtained for input signals (e.g. accelerograms) in the same manner. The Fourier Tool is described in more detail later in this paper.

Overview of Program Features

The main use of *NONLIN* is to perform response history analysis of a variety of simple structural systems. System behavior may be elastic or inelastic, and P-Delta effects may be included or excluded. Two basic system models are provided: a single degree of freedom model (SDOF) and a multiple degree of freedom model (MDOF). Details of the models are described in later sections of the paper. A third model, called the Dynamic Response Tool for proportionally damped systems (DRT-P) is available for explaining the concepts of natural modes shapes in linear multiple degree of freedom systems. A similar tool (DRT-N) for nonproportionally damped systems is also provided.

The models may be subjected to a variety of loads. Load type and applicability are summarized below:

Periodic load. This loading consists of a linear combination of sine, square, and triangular waves. The loading is automatically generated by the program, and may be applied as a force acting on the mass, or as an acceleration applied to the base of the structure. Applicable to SDOF and MDOF models. The DRT model may be subjected to sinusoidal harmonic load only.

Earthquake Loading. The program includes more than one hundred pre-recorded acceleration histories. New acceleration histories may be easily added. If the default characteristics of the ground motions are to be used, the ground motions may be accessed very rapidly through the QuickQuake feature. Applicable to SDOF and MDOF models.

Free Vibration. The structure may be subjected to an initial displacement and/or an initial velocity. Applicable to the SDOF model only. The free vibration analysis must be linear elastic.

Blast Loading. A short duration blast load may be computed internally, or a blast load record may be read from a text file. Applicable to SDOF model only.

Earthquake Ground Motion Processing

NONLIN provides a powerful suite of tools for processing earthquake ground motions. These tools may be used independently of the SDOF or MDOF solvers. A summary of the ground motion evaluation tools is provided below:

Input and Response History Plotting. A history plot is provided of the ground acceleration record and the accompanying velocity and displacement record, if available. If the velocity and displacement records are not available, *NONLIN* will produce them by integrating the ground acceleration record. High order polynomial base-line correction is provided upon such integration. A Fourier Amplitude Spectrum of any history plot may be obtained by a single mouse click.

Elastic Response Spectra. NONLIN computes all response spectra in real time using an "exact" piecewise linear methodology[2]. For a single ground motion, up to six spectra may be provided on one plot, representing different damping values. *NONLIN's* Incremental Dynamic Analysis tools allow the plotting of response spectra for several ground motions and a single damping value. The resolution of the

spectra (number of period values plotted) may be controlled by the user. Spectra may be plotted in tripartite (four way log), simple log, or arithmetic formats. Demand spectra (displacement on the horizontal axis and pseudoacceleration on the vertical axis) may also be plotted. A demand spectrum produced by *NONLIN* is shown in Figure 1. A variety of code spectra and other smoothed design spectra may be superimposed on any true ground motion spectrum.
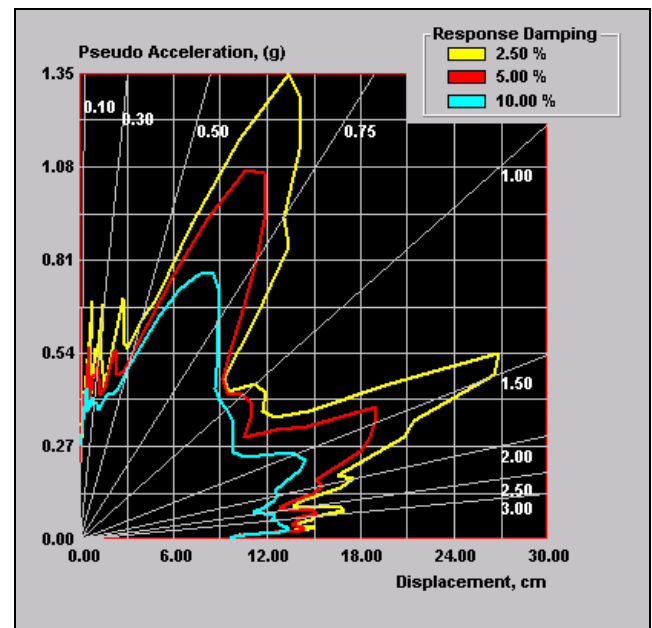

Figure 1. An Acceleration-Displacement Demand Spectrum.

Fourier Amplitude Spectra (FFT Tool). A Fourier amplitude spectrum may be obtained from almost any input or response history. A unique characteristic of the FFT tool is the ability to perform a "traveling" FFT, wherein the amplitude spectrum applicable to a certain window of time is shown together with spectra for the "prior" and "next" window. The FFT tool is illustrated in Figure 2.

Inelastic Response Spectra. A variety of inelastic "constant ductility" spectra may be computed for inelastic systems.
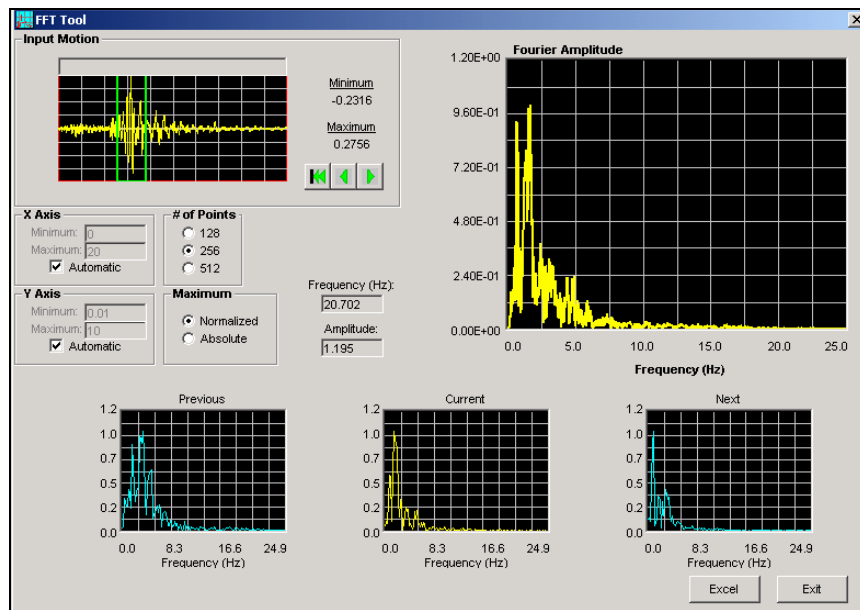
Figure 2. The traveling FFT Tool.

Earthquake Scaling Tools. The Incremental Dynamic Analysis feature of the SDOF model allows multiple ground motions to be scaled for design spectrum matching.

Single Degree of Freedom Model

The simplest system model in *NONLIN* is a single degree of freedom system, which may be described mathematically by the following equation of motion:

$$m\ddot{v}(t) + c\dot{v}(t) + R(t) + k_G v(t) = F(t)$$

where
  m is the mass of the system
  c is the damping constant
  R is the structural resisting force $= kv(t)$
      for linear systems
  k is the initial stiffness of the system
  k_G is the geometric stiffness
      (representing P-Delta effects)
  $F(t)$ is the applied load at time t
  $\ddot{v}(t)$ is the computed acceleration
  $\dot{v}(t)$ is the computed velocity, and
  $v(t)$ is the computed displacement

For earthquake problems, the equilibrium equation is rewritten as

$$m\ddot{v}(t) + c\dot{v}(t) + R(t) + k_G v(t) = -m\ddot{v}_g(t)$$

where $\ddot{v}_g(t)$ is the ground acceleration record and the displacement, velocity, and acceleration are relative to the base of the structure.

For the SDOF model the hysteretic behavior is bilinear, as shown in Figure 3. The positive and negative yield points are equal in magnitude, as are the positive and negative secondary (post-yield) stiffnesses. The system unloads on the same slope as the initial stiffness. P-Delta effects are handled by adding a linear spring with a negative stiffness, k_G, to the system.

The appropriate equilibrium equation is solved using the Newmark constant acceleration integration scheme[2]. The user may control the integration time step by setting a sub-stepping parameter. Due to the use of a very small time step, no iteration is performed. The accuracy of the solution may be checked through the use of an energy balance.

The actual form for entering the system parameters is shown in Figure 4. Note that the cells in the figure indicate the current state of
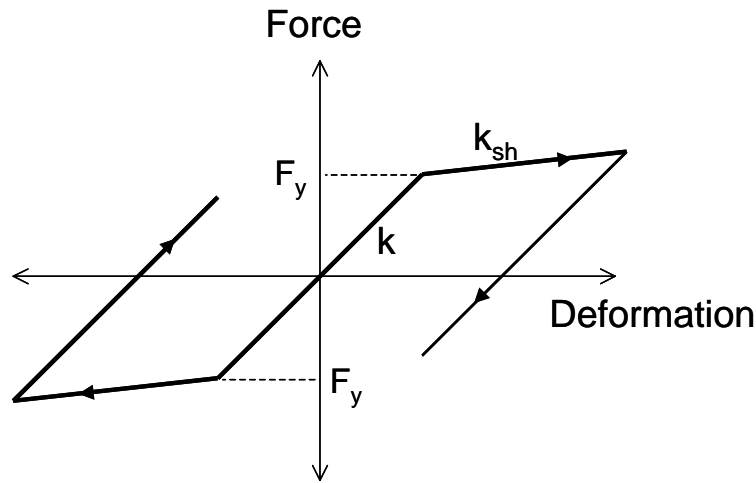
Figure 3. Bilinear Force-Deformation Relationship Used in SDOF Model.
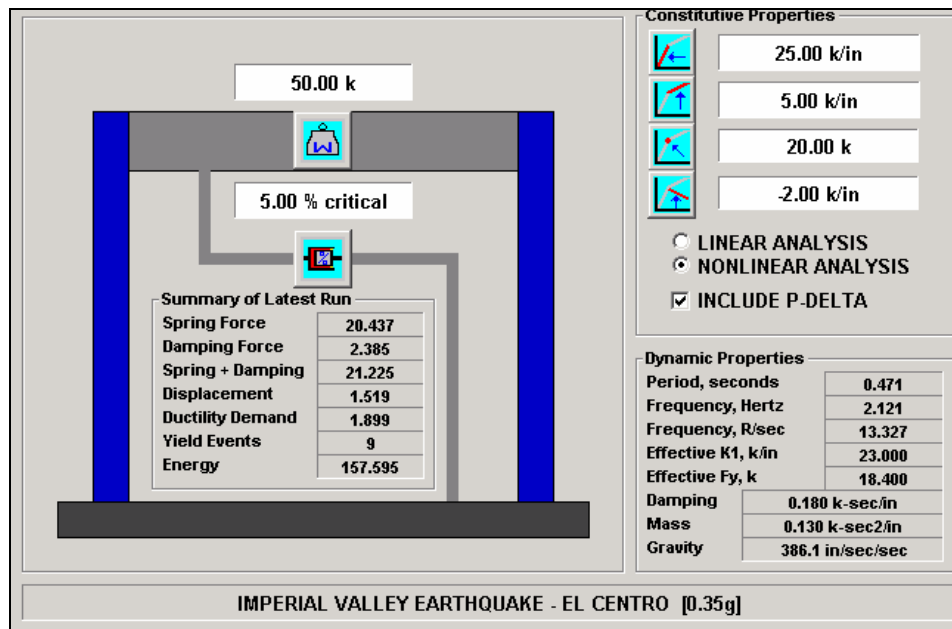

Figure 4. SDOF Data Entry Form (Showing summary of results).

the problem. In this case, the structure was analyzed using the Imperial Valley earthquake.

A variety of results are available after the solution has been performed. These include response history plots, X-Y (force-displacement plots), animations, and energy plots. An energy plot, showing a history of strain+kinetic energy

(yellow), damping energy (green), and hysteretic energy (red), is provided in Figure 5. The energy plot is very useful when teaching the concepts of added damping in structures, where it is shown that the amount of the red area, representing damage, is significantly reduced when damping is increased.
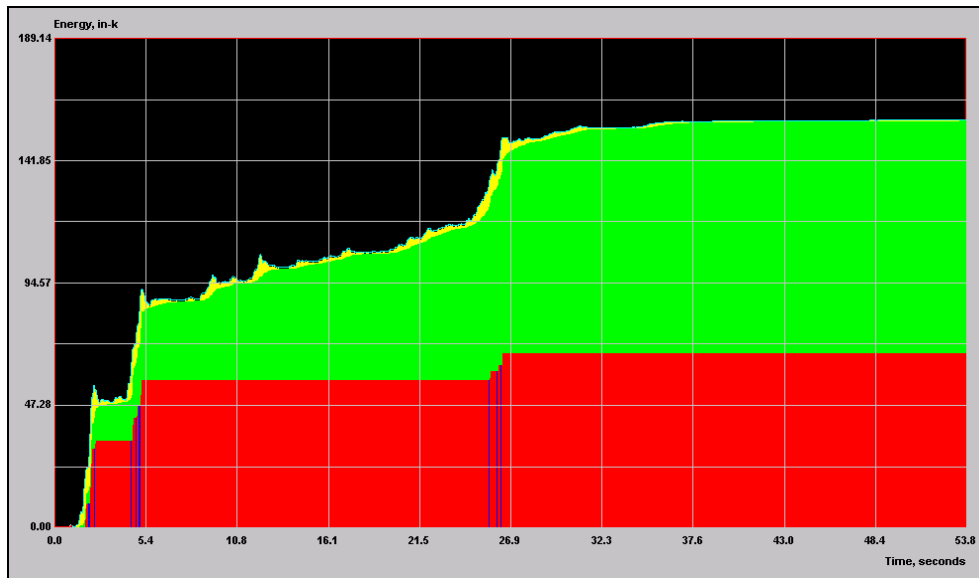
Figure 5. Energy Plot Produced by the SDOF Solver.

Incremental Dynamic Analysis

An important new feature in Version 7.0 of *NONLIN* is the Incremental Dynamic Analysis (IDA) tool. IDA is an analysis procedure in which a particular structure is repeatedly subjected to the same ground motion, with the ground motion intensity increasing for each analysis. A plot of some response parameter (say total drift) versus earthquake intensity is called a single ground motion IDA plot. IDA plots for several earthquakes may be plotted together. The conceptual basis of IDA has been presented in detail by[9].

When multiple ground motion IDA is performed, it is necessary to scale the ground motions in some consistent manner. Shome et al[6] have shown that scaling each of the ground motions to produce spectral pseudoaccelerations equal to the same target acceleration produces minimum dispersion among the results. When scaling, all pseudo accelerations are computed at the structure's own natural period and inherent and damping.

NONLIN's IDA tool allows the user to select up to seven ground motions, and automatically scales the ground motions to the criterion stated above.   Other scaling (such as equal peak

ground acceleration) may also be performed. The IDA is then performed for each earthquake in sequence. Up to 40 individual increments of earthquake intensity may be used for each ground motion. Figure 6 shows *NONLIN*'s IDA form after a structure has been analyzed using seven ground motions with twenty intensity increments per ground motion.

IDA is a very useful tool for understanding seismic behavior, and most particularly for evaluating the effect of systemic parameter changes on response[3].   Among the most unusual behaviors exhibited by IDA is structural "resurrection", where a system collapses under a ground motion of a particular intensity, and survives earthquakes with larger intensities. Results of this type illustrate the highly chaotic nature of seismic response, and reinforce the effect of uncertainty and the importance of probabilistic approaches in performance based earthquake engineering.

Multi Degree of Freedom Model

The second major model provided by *NONLIN* is the MDOF model, which consists of a base isolated one story frame. A chevron brace may be used alone, or as a means for incorporating a passive energy device into the system. The
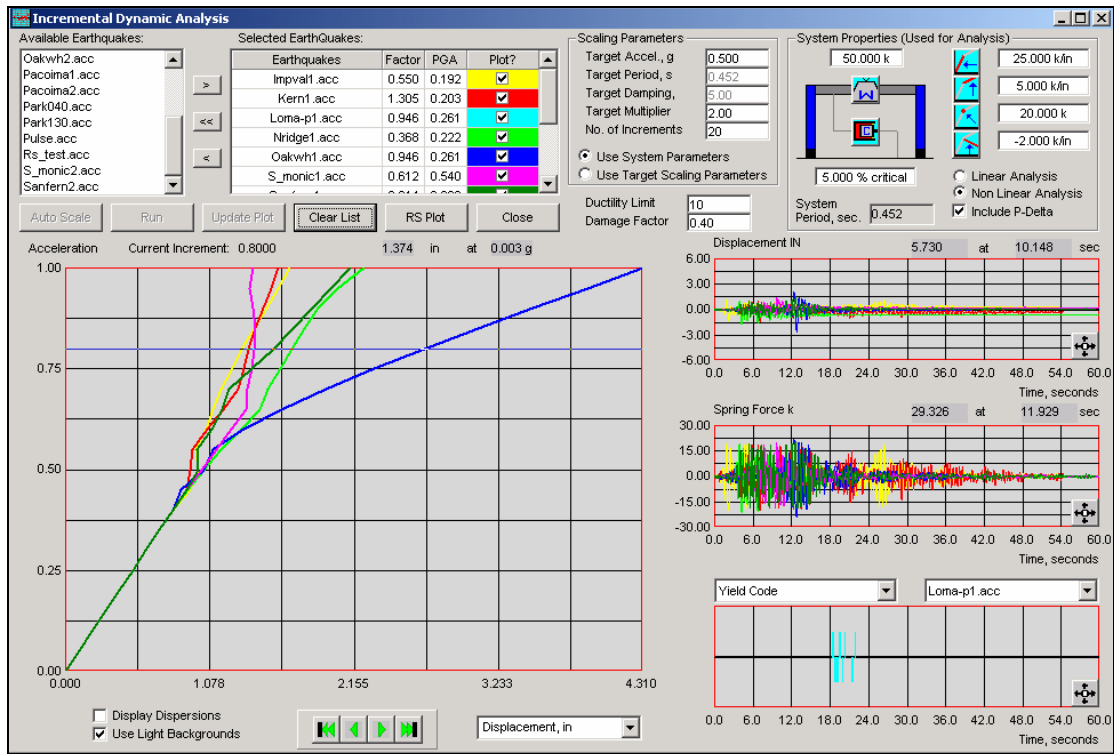
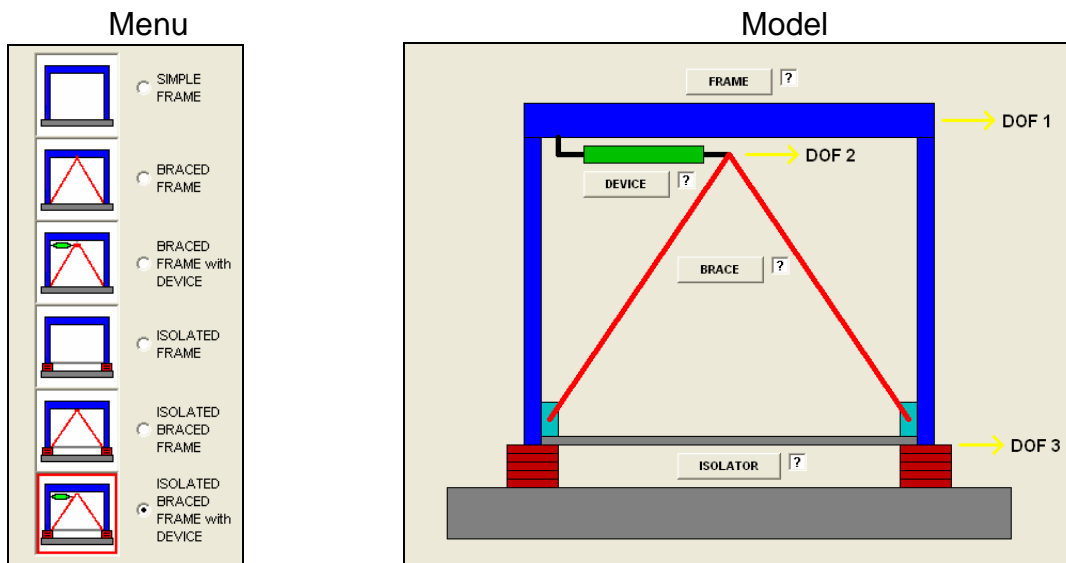Figure 6. The IDA Form and Results for Seven Ground Motions.

Figure 7. The NONLIN MDOF Models.

model which incorporates all of the features described above is illustrated at the right of in Figure 7. The menu of available systems that incorporate one of more of the features is shown at the left of Figure 7. The user selects the appropriate model by clicking on the appropriate diagram in the menu. The maximum number of degrees of freedom for the model is three: (DOF 1) representing the "roof" displacement, DOF 2 representing the lateral displacement at the top of the brace, and DOF 3 representing the displacement at the top of the base isolator. The deformation in the device at any point in time is taken as DOF 1 – DOF 2.

The system of equations solved for the MDOF model under earthquake excitation is:

$$M\ddot{v}(t) + C\dot{v}(t) + Kv(t) = -MR\ddot{v}_g(t)$$

where $v(t)$ and its time derivatives are vectors of nodal displacement, velocity, and acceleration. R is simply a vector of ones. The mass matrix M, damping matrix C, and stiffness matrix K are always of size 3-by-3. If desired, P-Delta effects are explicitly included in K. Internal parameters are set appropriately high or low to emulate a one or two-degree of freedom model.

When incorporated, the brace is assumed to remain elastic. A wide variety of hysteretic behavior may be assigned to the isolator, frame, or device. The models may be multilinear or curvilinear, and may incorporate degrading stiffness, degrading stiffness, or pinching. The coding for the hysteretic model was provided by Sivaselvan and Reinhorn[7], and was incorporated into *NONLIN* as a Fortran Dynamic Link Library. Figure 8 illustrates the *NONLIN* data input frame that is used to specify the hysteretic model. Note that the model may be "tested" by a cyclic load before it is incorporated into the system model. This is shown in the right hand panel of Figure 8.

The device may be set up as a linear or nonlinear viscous dashpot, with a force-velocity relationship given by $F_{Device} = C \times signum(\dot{v}_R) \times |\dot{v}_R|^\alpha$ where C is the damping coefficient of the device, $\dot{v}_R$ is the deformational velocity in the device, and α is an exponent ranging from about 0.2 to 2.0. Low values of α represent systems with "softening" behavior and high values of α represent hardening behavior. When $\alpha = 1.0$ the dashpot is linear.

The equations of equilibrium for the MDOF model are solved using the Newmark constant acceleration scheme with a constant user specified integration time step. As stated earlier, the three degree of freedom system is solved regardless of the configuration. Iteration is not performed. Desired accuracy may be obtained by utilizing a very small time step. Output for the MDOF tool consists of response histories of the various degrees of freedom, as well as force-displacement and force-velocity plots for the various structural components.

In an educational environment, the MDOF tool is particularly effective in demonstrating the benefits of base isolation and passive energy. The effect of degrading strength and degrading stiffness on system response may also be studied in some detail.

Dynamic Response Tool (Proportionally Damped Models)

The third analysis tool in *NONLIN* is the Dynamic Response Tool for Proportionally Damped Systems (DRT-P). This tool was specifically designed to illustrate the concepts of natural mode shapes and natural frequencies. The effects of modal damping may also be demonstrated. The DRT-P tool is illustrated in Figure 9.
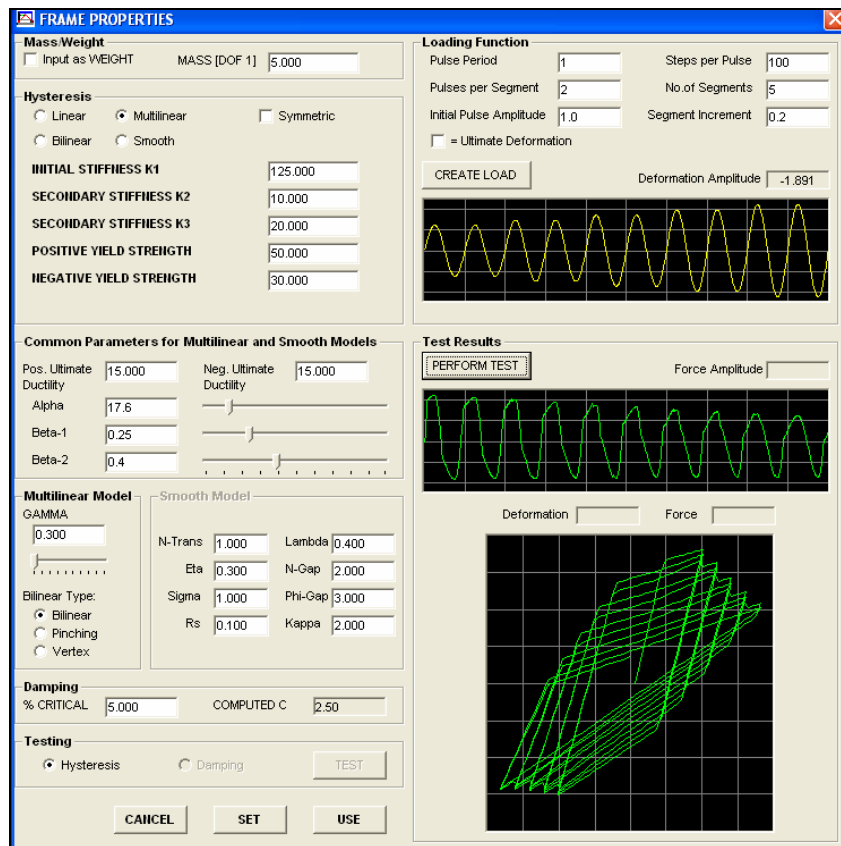
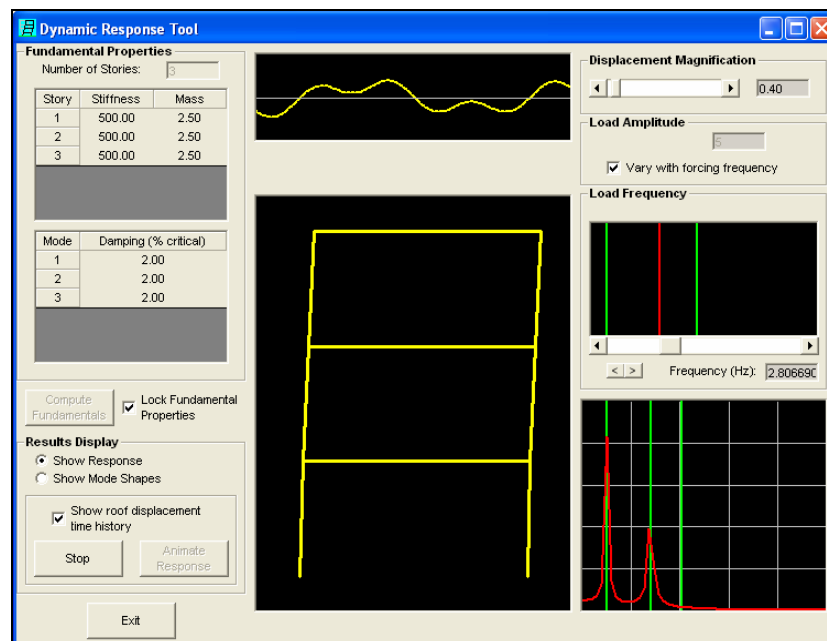Figure 8. The Hysteretic Property Selection Tool in the MDOF Model.

Figure 9. The Dynamic Response Tool (Proportional Damped Systems).

The structural system in the DRT-P is a multi-level linear elastic "shear building". The mass and stiffness is specified for each level, and a damping ratio for each mode is specified. After the properties are entered, the program computes and displays the undamped mode shapes and frequencies. The mode shapes may be animated, showing real time variations in the amplitude of each mode (the animated modes vibrate at "true" frequency).

After the mode shapes are computed, the structure is analyzed for a system of harmonic loads, with the same harmonic loading function applied at each level. The user may specify only the frequency of the harmonic load, but the frequency may be changed at any time during the analysis. The system is analyzed using modal superposition, with each mode's contribution being calculated by the piecewise exact integration method.

If the harmonic load is applied at one of the structure's natural frequencies, the structure will resonate and will eventually begin to vibrate in the mode shape corresponding to the frequency. The vibrating shape of the structure is continuously displayed, together with the response history trace at the top of the structure (shown above the structure in Figure 9), and a Fourier Amplitude Spectrum of the roof displacement trace (shown to the right of the structure in Figure 9). If the loading frequency is changed, say from the first mode frequency to the second mode frequency, the first mode response eventually dies out (due to damping) and the second mode shape eventually emerges as the dominant vibrating shape. Changing now to a loading at the third modes' natural frequency, the first mode response reappears due to the introduction of transients, the first and the second mode components of displacement damp out, and the third mode response becomes dominant.

One of the most useful aspects of the DRT is the ability to change the individual modal damping ratios and then to demonstrate the effect. For example, if the first mode damping

ratio is increased to 10% and the remaining modes are left at 2%, the first mode response will be seen to decay much more rapidly than when the damping in the first mode was 2% critical.

Dynamic Response Tool
(Nonproportionally Damped Models)

The fourth analysis tool in *NONLIN* is the Dynamic Response Tool for Nonproportionally Damped Systems (DRT-N). This tool was specifically designed to illustrate the concepts of complex eigenvalues and mode shapes, and to "demystify" the use of complex numbers in engineering calculations. The DRT-N tool is illustrated in Figure 10.

As may be seen in the figure, the DRT-N tool is similar to the DRT-P tool, except that the story damping constants *C* are explicitly provided, thereby allowing the analysis of nonproportionally damped systems. Once the system properties are input, the eigenvalues and eigenvectors are computed. The eigenvalues are decomposed into to the frequency and damping ratio information they contain, and the eigenvectors are decomposed into amplitude and phase. This decomposition illustrates that complex numbers are really nothing more than a convenient notation, wherein one quantity can store two pieces of information.

Also contained on the tool are animated mode shapes (center panel), animated displacement histories for the vibrating shapes (top panel), and animated plots of the coordinates of the complex eigenvectors (upper right panel). Animation of the deflected shape for nonproportionally damped systems illustrates the interesting phase relationships that occur in the mode shapes[1]. One good use of the tool is the illustration of the difference between proportionally damped systems (everything in phase) and nonproportionally damped systems (everything mostly out of phase). It may be shown that the higher modes generally contain larger phase differences than the lower modes,
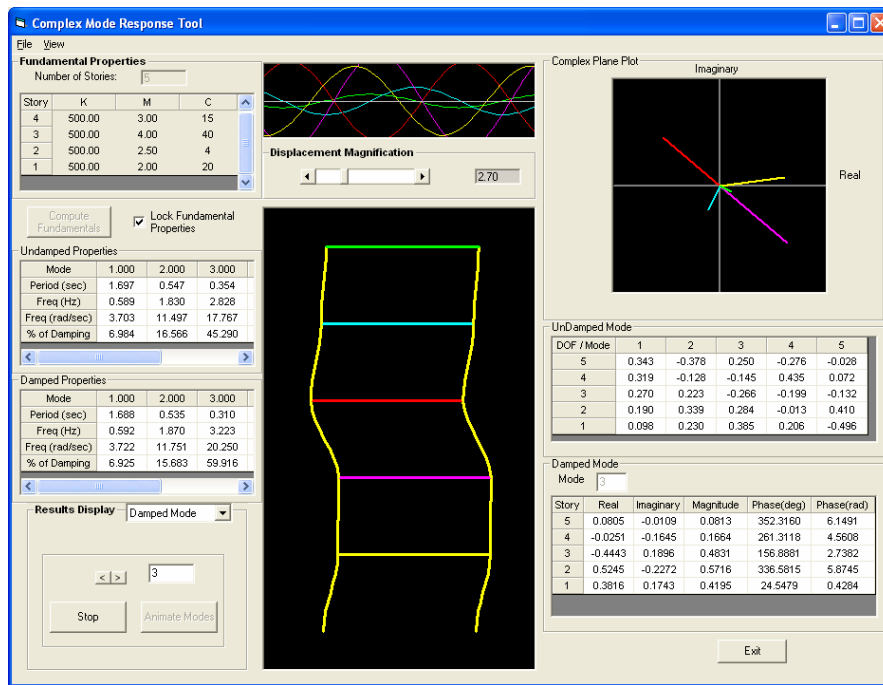
Figure 10. The Dynamic Response Tool (Nonproportional Damped Systems).

and that the more nonproportionally damped a system is, the greater the phase differences.

Future Modifications

Several modifications are underway for *NONLIN Version 8.* For the SDOF model, an improved integration scheme is under development that includes Newton-Raphson iteration for elimination of equilibrium unbalances. A variety of more complex hysteretic models are also being provided, including those developed by Sivaselvan and Reinhorn[7]), and by Ibarra[4]. The elastic response spectra routine will be modified to produce true velocity and true total acceleration spectra for comparison with the corresponding pseudo quantities. Finally, the large ground motion database provided by *EQTOOLS*[8] will be made available to *NONLIN*.

The IDA tool is being modified to perform variation of parameter IDA, wherein a family of IDA curves will be produced for a system subjected to a particular ground motion, and

where a certain system parameter (e.g yield point) will be systematically varied.

The MODF tool is being updated to provide a larger variety of hysteretic models, an increased number of stories (three levels plus one base isolation level), improved response plots, animation of the deflected shape, and inclusion of vertical accelerations. Energy histories for each component and for the entire structure will be displayed.

Classroom Use

At Virginia Tech, *NONLIN* is used in a graduate course entitled Structural Design for Seismic Effects. Homework problems are assigned as follows:

1) Use *NONLIN* to verify MATCHAD or published solutions of free vibration or harmonic load response of single degree of freedom systems

2) Use *NONLIN* to develop and compare true ground motion response spectra with code based response spectra.

3) Use *NONLIN* to verify the "Equal Displacement" principle of earthquake resistant design, where it is assumed that computed displacements for yielding systems are approximately equal to displacements computed for linear elastic systems with the same stiffness.

NONLIN is also used in a variety of continuing education courses, for example, the *ASCE Fundamentals of Earthquake Engineering* course. Here, the program is used to demonstrate the principles of structural dynamics, and particularly, modal decomposition of multiple degree of freedom systems. The program was also used extensively in the preparation of the course materials.

Program Availability

NONLIN, as well as other tools for earthquake engineering education, may be downloaded at no cost from the web site celes.ictas.vt.edu/research/downloads. Other software on the site includes *EQTools*[8] and *NONLIN Pro*, a graphical pre and post-processor for the well-know nonlinear analysis program Drain-2D[5].

Acknowledgements

References

1. Charney, F.A., and McNamara, R.J. (2002), "Use of Damped Mode Shapes in Visualizing the Efficiency of an Auxilliary Damped 39-Story Office Building", Proceedings of the 7th National Conference in Earthquake Engineering, Boston MA.

2. Chopra, A.K. (2001), *Dynamics of Structures: Theory and Applications to Earthquake Engineering.* Prentice Hall: Englewood Cliffs, NJ.

3. De, Samrat (2004), *Effect of Variation of The Systemic Parameters on The Structural Response of Single Degree of Freedom Systems Subjected to Incremental Dynamic Analysis*, M.S. Thesis, Department of Civil Engineering, Virginia Tech, Blacksburg, VA.

4. Ibarra, Luis F. (2003). *Global Collapse of Frame Structures Under Seismic Excitation*, Ph.D. Dissertation, Department of Civil and Environmental Engineering, Stanford University, Palo Alto, CA.

5. Prakesh, V. and Powell, G. H., (1993). DRAIN 2D-X Users Guide, University of California, Berkeley, California.

6. Shome, N., Cornell, C.A., Bazzurro, P., and Carballo, J.E. (1998), "Earthquakes, Records and Nonlinear Responses", *Earthquake Spectra*, 14, 3, pp. 469-498.

7. Sivaselvan, M.V. and Reinhorn, A.M. (1999), "Hysteretic models for cyclic behavior of deteriorating inelastic structures", *Technical report MCEER-99-0018*, State University of New York at Buffalo.

8. Syed, Riaz (2004), *Development of Computational Tools for Evaluation of Strong Motions within a Performance-Based Seismic Design Framework*, M.S. Thesis, Department of Civil Engineering, Virginia Tech, Blacksburg, VA.

9. Vamvatsikos, D. (2002), *Seismic Performance, Capacity, and Reliability of Structures as Seen Through Incremental Dynamic Analysis*, Ph.D. Dissertation, Department of Civil and Environmental Engineering, Stanford University, CA.

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