CROSSFLOW AIR-LIQUID HEAT EXCHANGER RATING AND DESIGN USING EXCEL

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Abstract

Spreadsheets are often the preferred computational tool used by many engineers, despite the capabilities of packages like Matlab, Mathcad, and TKSolver. This paper uses Excel to calculate the heat transfer and pressure drops in a crossflow air-liquid heat exchanger (HX) of known dimensions, i.e., rate the HX. Visual Basic functions needed for these calculations are described. The paper then demonstrates the use of this rating process in conjunction with Excel's simultaneous nonlinear equation solver to design a HX - given mass flow rates, inlet temperatures, specified heat transfer rate, and maximum allowable pressure drops.

Introduction

Problem

An unfinned crossflow heat exchanger (HX) with aligned (vs staggered) tubes is shown in Figure 1. A surface like this one is to be used to deliver a specified heat transfer rate subject to specified maximum pressure drops for both fluid streams. For given values of the fluid mass flow rates, inlet temperatures, tube spacings S_T, S_L, and tube diameter D, the following can be varied to satisfy the performance specifications:

- Number of transverse tube rows N_T. $N_T = 3$ in Figure 1.
- Number of longitudinal tube rows, N_L. $N_L = 4$ in Figure 1.
- Tube length L_{tube}.
- The number of tubes manifolded together N_m, i.e., the number of tubes through which the tubeside fluid is distributed. In Figure 1, $N_m = 3$.

We will use Excel to calculate the performance of a HX, i.e., rate the HX, for a given set of parameters N_T, N_L, L_{tube}, and N_m and then use Excel's solver to determine the values that will satisfy the performance specifications.

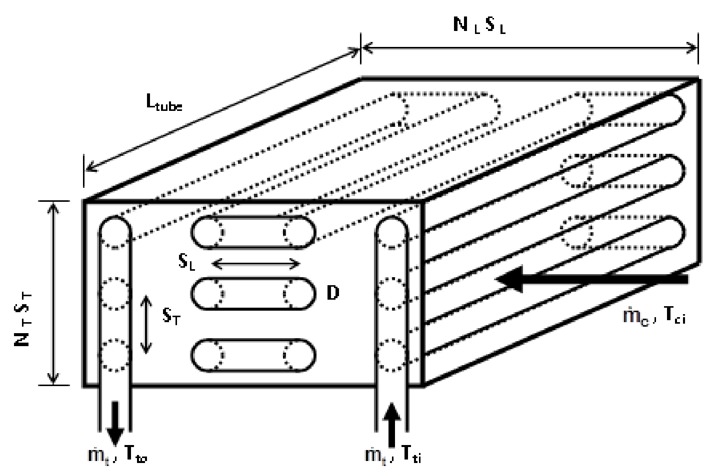

Figure 1. Crossflow Heat Exchanger – Unfinned.

Note that the number of tubes is $N_{tube} = N_T N_L$ and that the number of times the tubeside fluid travels the length of the HX is $N_{pass} = N_{tube}/N_m$. In Figure 1, $N_{tube} = 12$ and $N_{pass} = 4$.

Given

- Desired heat exchange rate q_{spec}.
- Maximum allowable pressure drops in the tubes, $\Delta P_{t,\ spec}$, and for the crossflow fluid, $\Delta P_{c,\ spec}$.
- Mass flow rates of the tubeside fluid and the crossflow fluid, $\dot{m}_t$ and $\dot{m}_c$.
- Inlet fluid temperatures T_{ti} and T_{ci}.
- Tube inside and outside diameters, D_i and D_o, and thermal conductivity k_t.
- Tube spacing transverse to the flow direction of the crossflow fluid, S_T, and the spacing in the longitudinal direction, S_L. Aligned or staggered tube spacing is specified.

- If there are fins, fin thickness, δ_f, and thermal conductivity k_f.
- If the fins are individual round fins or spiral fins, fin outer diameter D_f.
- Head loss coefficients for connecting piping on the tubeside.
- Fouling factors.
- The following properties for the crossflow surface, given (e.g., surfaces in Kays and London [1]) or calculated:
  - Hydraulic diameter, D_h.
  - Heat transfer surface area per volume of heat exchanger, β.
  - Ratio of minimum cross-sectional flow area, $A_{cr,\ min}$, to frontal area, A_{fr}, of the heat exchanger, σ.
  - The ratio of fin surface area to total heat transfer area A_{fin}/A.

Solution

Functions Required

Rating the HX requires the following Visual Basic functions that can be called from the spreadsheet:

- Fanning friction factor, f_F, for both streams as function of Re. For finned crossflow surfaces, the manufacturer often supplies a plot that a user can curve fit (See example below). For flow in tubes, Churchill's method[2] is used for this paper.
- Nusselt number, Nu, for both streams as function of Re and Pr. Again, for finned crossflow surfaces, the manufacturer often supplies a plot that a user can curve fit. In this paper, f_F, the method in[2] is used in the Petukhov equation[3] for flow in tubes.
- Fin efficiency, η_{fin}, as a function of heat transfer coefficient, fin thermal conductivity, tube outer diameter, and tube spacing (or fin diameter if individual/spiral fins). In this paper, Schmidt's empirical expressions[4] are used.
- HX effectiveness, ε, as function of transfer units Ntu, ratio of mass flow rate-specific heat products, $C^* = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$, and heat exchanger type, i.e., parallel, counterflow, etc. These functions are available in Kays and London[1] and most heat transfer texts.

Design Process

Set up a spreadsheet to execute the following procedure:

1. Estimate the specific heat C_p for both streams and use the first law of thermodynamics to estimate the outlet temperatures T_{to} and T_{co} for the specified heat exchange rate q_{spec}. Or, calculate q_{spec} and the unknown outlet temperature if T_{to} or T_{co} is specified.

2. Get the inlet density for both streams, ρ_{ti} and ρ_{ci}, at the inlet temperatures and the outlet densities, ρ_{to} and ρ_{co}, at the outlet temperatures.

3. At the mean temperatures $(T_{ti} + T_{to})/2$ and $(T_{ci} + T_{co})/2$ for each fluid, get the following fluid properties for both streams:

- Dynamic viscosity μ
- Density ρ
- Specific heat C_p
- Thermal conductivity k
- Prandtl number Pr

4. Estimate the crossflow frontal area $A_{fr,c} = N_T S_T L_{tube}$, N_L, and N_m. A procedure for estimating these is given later. Note that there are many combinations of N_T and L_{tube} that will give the same $A_{fr,c}$. This gives the designer some flexibility in choosing the height ($N_T S_T$) and width (L_{tube}) combination to achieve the same frontal area for the HX.

5. Rate the HX using the estimated values of N_L, N_m, and $A_{fr,c}$ to calculate output cells q_{calc}, $\Delta P_{t,\,calc}$, and $\Delta P_{c,\,calc}$.
   a. Calculate mass velocity of the crossflow fluid

$$G_c = \frac{\dot{m}_c}{\sigma \cdot A_{fr,c}} \qquad (1)$$

   b. Calculate mass velocity of the tubeside fluid

$$G_t = \frac{\dot{m}_t}{N_m \cdot \pi D_i^2} \qquad (2)$$

   c. Calculate Re = GD_h/μ for both fluids. Note that $D_h = D_i$ for the tubeside fluid.
   d. Use your functions to calculate Nusselt number Nu (Re, Pr) and Fanning friction factor f_F (Re) for both fluids.
   e. Calculate heat transfer coefficient h = $Nu \cdot k/D_h$ for both fluids.
   f. If the tubes are finned, use your function for fin efficiency to calculate η_{fin} using h for the crossflow fluid, fin dimensions, and fin thermal conductivity.
   g. Calculate total area effectiveness

$$n_T = (1 - A_{fin}/A) + \eta_{fin} A_{fin}/A \qquad (3)$$

   for the crossflow fluid. Note that A_{fin}/A = 0 and n_T = 1 if there are no fins.
   h. Calculate heat transfer areas for both fluids.

$$A_t = L_{tube} N_T N_L \pi D_i \qquad (4)$$

   where $L_{tube} N_T = A_{fr,c}/S_T$ $A_c = \beta A_{fr,c} N_L S_L$.
   i. Calculate UA using

$$\frac{1}{UA} = \frac{1}{(hA)_t} + \frac{R''_{ft}}{A_t} + \frac{\ln(D_o/D_i)}{2\pi N_L N_T L_{tube} k_t} + \frac{R''_{fc}}{(\eta_T A)_c} + \frac{1}{(\eta_T hA)_c} \qquad (5)$$

   where R''_{ft} and R''_{fc} are tubeside and crossflow fouling factors.
   j. Calculate

$$Ntu = UA/(\dot{m}C_p)_{min} \qquad (6)$$

$$C^* = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max} \qquad (7)$$

   Use your function of Ntu, C*, and HX type to get HX effectiveness, ε.
   k. Calculate heat exchange rate

$$q_{calc} = (\dot{m}C_p)_{min} \cdot \varepsilon \cdot |T_{ti} - T_{ci}| \qquad (8)$$

   l. Calculate pressure drops for both fluids, $\Delta P_{t,calc}$ and $\Delta P_{c,calc}$, using (Kays and London [1])

$$\Delta P = \frac{G^2}{2\rho_i}\left[\left(K_c + \frac{\rho_i}{\rho_o}K_e + \frac{\rho_i}{\rho}\Sigma K + \frac{\rho_i}{\rho} f_F \frac{A_w}{A_{cr,min}}\right) + \left(\frac{\rho_i}{\rho_o} - 1\right)(1+\sigma^2)\right] \qquad (9)$$

   for both fluids, where K_c and K_e are contraction and expansion loss coefficients and ΣK accounts for other fittings, e.g., U-bends in the tubes at the ends of a heat exchanger like those shown in Figure 01. The wetted area A_w is the same as the heat transfer area A for each stream.

6. Calculate ($\Delta P_{c,spec}$ - $\Delta P_{c,calc}$), ($\Delta P_{t,spec}$ - $\Delta P_{t,calc}$), and (q_{spec} - q_{calc}) in separate cells.

7. Use Excel's solver to drive the cells containing (q_{spec} - q_{calc}), ($\Delta P_{c,spec}$ - $\Delta P_{c,calc}$), and ($\Delta P_{t,spec}$ - $\Delta P_{c,calc}$) to zero by changing the cells containing $A_{fr,c}$, N_{gr}, and N_L. (Note the mathematics - 3 equations, 3 unknowns.)

8. Round the resulting N_T and N_L up or down to whole numbers and modify L_{tube} as needed to achieve q_{spec} while satisfying constraints for $\Delta P_{t,\,spec}$ and $\Delta P_{c,\,spec}$.

Procedure for estimating initial values of N_T, N_L, N_m, and L_{tube}

The procedure given by Shah [5] is used here, with modification for finned tube crossflow heat exchangers.

1. Calculate C* (Eq. 7) and effectiveness

$$\varepsilon = q_{spec}/[(\dot{m}C_p)_{min} \cdot | T_{ti} - T_{ci}|] \qquad (10)$$

and determine the required number of transfer units Ntu.

2. Estimate $ntu_{one\ side} = [(\eta_T hA)/(\dot{m}C_p)]$ for each of the two fluids.

$ntu_{one\ side}$
= 2 Ntu for a gas-gas or liquid-liquid heat exchanger (11a)
= 1.1 Ntu for the gas side of gas-liquid heat exchanger (11b)
= 10 Ntu·C* for the liquid side of gas-liquid heat exchanger (11c)

3. Estimate crossflow fluid Colburn j-factor j_c and use it to estimate N_L as

$$N_L = \frac{ntu_c \, Pr_c^{2/3}}{4\eta_T j_c (S_L / D_h)} \qquad (12)$$

Round to a whole number. Use j_c = 0.008 unless a better value is available. Estimate η_T = 0.8 for a finned surface. η_T = 1 for an unfinned surface.

4. Estimate N_m as

$$N_m = \frac{4}{\pi D_i^2}\sqrt{\frac{ntu_t \, Pr_t^{2/3}}{(j/f_F)_t}} \cdot \frac{\dot{m}_t}{\sqrt{2\rho_t \Delta P_{t,spec}}} \qquad (13)$$

Round up to a whole number. Use $(j/f_F)_t$ = 0.5 unless a better value is available. Note that if the tubeside fluid is distributed to all the tubes, the resulting $N_m = N_T N_L$, and $N_T = N_m/N_L$.

5. Estimate the crossflow frontal area $A_{fr,c}$.

$$A_{fr,c} = N_T S_T L_{tube}$$
$$= \frac{1}{\sigma_c}\sqrt{\frac{ntu_c \, Pr_c^{2/3}}{(j/f_F)_c}} \cdot \frac{\dot{m}_c}{\sqrt{2\rho_c \Delta P_{c,spec}}} \qquad (14)$$

where $(j/f_F)_c$ is a ballpark value for the finned surface. Use 0.3 unless a better value is available. If the tubeside fluid is distributed to all the tubes, $N_T = N_m/N_L$ and $L_{tube} = A_{fr,c}/N_T S_T$.

Example

Problem

The objective is to design, using the compact finned surface shown in Fig. 2, a hot water coil that will heat 14400 lbm/hr of 40 F air flowing over the tubes to 105 F using 7752 lbm/hr of 180 F water flowing in the tubes. The water pressure drop cannot exceed 4 psi, and the air pressure drop cannot exceed 0.7 in. WG. The tubes are copper, and the fins are aluminum. The headers used to connect the water piping to the coil cause contraction and expansion losses of K_c = 3 and K_e = 5. As shown in Figure 1, U-shaped bends are needed at the end of each tube to connect it to another tube. The number of these U bends required per pass is N_{tube}/N_m - 1, and K for each U-bend is 0.9. Dimensions and other properties of the surface are shown below in Table 1. The

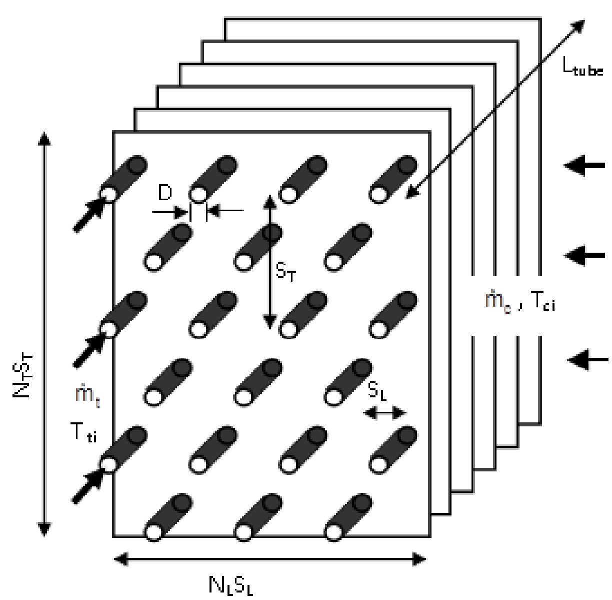

Figure 2. Finned Crossflow Exchanger.

Fanning friction factor and Nusselt number for this surface are calculated using curve fits of data supplied by the surface manufacturer

$$f_F = 0.08387\,Re^{-0.2075}$$

$$Nu = 0.1019\,Re^{0.6407}\,Pr^{1/3}$$

Table 1 - Surface Properties

Property	Water t	Air c	units
tube alignment - staggered			
transverse tube spacing, S_T		1.50	in.
longitudinal tube spacing, S_L		1.75	in.
tube conductivity, k_t		220	Btu/hr-ft-F
heat transfer area per volume of heat exchanger, β		169	ft^2/ft^3
minimum internal flow area/frontal area, σ		0.481	
fin area/total heat transfer area	0	0.950	
air side hydraulic diameter, D_h		0.1368	in.
tube inside diameter, D_i	0.576		in.
tube outside diameter, D_o		0.676	in.
fin conductivity, k_f		118	Btu/hr-ft-F
fin thickness, t_f		0.01	in.
fouling factor	0	0	hr-F-ft^2/Btu

Solution

We will (1) estimate the dimensions of the heat exchanger, (2) rate the heat exchanger with those dimensions, and then (3) use Excel's solver to correct the dimensions to satisfy the heat transfer rate requirement and pressure drop constraints.

Application of the first law using values of specific heat in Table 2 below gives the required heat transfer rate q_{spec} = 2.25E+05 Btu/hr and water outlet temperature T_{to} = 151.0 F. Average fluid properties based on these temperatures are shown in Table 2.

Table 2 - Fluid Properties

Property	Water t	Air c	units
mass flow rate, $\dot{m}$	7752	14400	lbm/hr
specified max pressure drop, ΔP_{spec}	4.0	0.7	psi, in. WG
inlet temperature, T_i	180	40	F
outlet temperature, T_o	151.0	105	F
density, ρ	61.1	0.076	lbm/ft^3
specific heat, C_p	1.00	0.24	Btu/lbm-F
$\dot{m}\,C_p$	7752	3456	Btu/hr-F
dynamic viscosity, μ	9.70E-01	4.40E-02	lbm/ft-hr
Prandtl number, Pr	2.53	0.71	
conductivity, k	0.3840	0.0148	Btu/hr-ft-F

At the needed heat transfer rate of q_{spec} = 2.25E+05 Btu/hr, the heat exchanger effectiveness from Eq. 10 is ε = 0.464. For a both sides unmixed crossflow heat exchanger with C* = $(\dot{m}C_p)_c/(\dot{m}C_p)_t = 0.446$ and ε = 0.464, Ntu = 0.741.

Estimate Dimensions. To estimate heat exchanger dimensions, additional assumed values shown in Table 3 are needed.

Table 3 - Assumed Additional Values

Property	Water - t	Air - c
transfer units-one side, ntu	3.30	0.81
Colburn j-factor, j		0.008
j/f_F	0.5	0.3
total area effectiveness, η_T		0.8

The ntu values were estimated using Eqs. 11b and 11c, i.e., ntu_c = 1.1 Ntu and ntu_t = 10·C*·Ntu.

Using the values in Tables 1 - 3 in Eqs. 12 - 14 gives the following results for estimated heat exchanger dimensions:

$$N_L = \frac{ntu_c\,Pr_c^{2/3}}{4\eta_T\,j_c(S_L/D_h)} = 1.98$$

$$N_m = \frac{4}{\pi D_i^2}\sqrt{\frac{ntu_t\,Pr_t^{2/3}}{(j/f_F)_t}}\cdot\frac{\dot{m}_t}{\sqrt{2\rho_t\Delta P_{t,spec}}} = 2.77$$

$$A_{fr,c} = N_T S_T L_{tube}$$
$$= \frac{1}{\sigma_c}\sqrt{\frac{ntu_c\, Pr_c^{2/3}}{(j/f_F)_c}}\cdot\frac{\dot{m}_c}{\sqrt{2\rho_c \Delta P_{c,spec}}} = 3.24\,ft^2$$

The values of N_L and N_m must be rounded. In this case, N_L = 2 and N_m= 3 will be used. There are many combinations of tube length L_{tube} and transverse rows N_T that will provide the estimated air-side frontal area of $A_{fr,\,c}$ = 3.24 ft^2. A value of N_T = 8 will also be assumed, giving an estimated heat exchanger height = $N_T S_T$ = 1.0 ft and tube length of $L_{tube} = A_{fr,\,c}/(N_T S_T)$ = 3.24 ft.

Rate Heat Exchanger. The next step is to rate a heat exchanger constructed of the surface above with the dimensions N_L = 2, N_m = 3, N_T = 8, L_{tube} = 3.24 ft. The analysis produced the following results:

- q_{calc} = 1.70E+05 Btu/hr (Eq. 8) (q_{spec} = 2.25E+05 Btu/hr)
- $\Delta P_{c,\,calc}$ = 0.46 in. WG (Eq. 9) ($\Delta P_{c,\,spec}$ = 0.7 in. WG)
- $\Delta P_{t,\,calc}$ = 5.3 psi (Eq. 9) ($\Delta P_{t,\,spec}$ = 4.0 psi)

The heat transfer rate is less than the specified value. The air-side pressure drop is less than the specified maximum, while the water-side pressure drop exceeds the specified maximum.

Adjust Dimensions. Since the estimated dimensions do not satisfy the performance specifications, values of N_L, N_m, and $A_{fr,\,c}$ must be adjusted. To satisfy the specifications, use Excel's solver to drive cells containing (q_{spec} - q_{calc}), ($\Delta P_{c,\,spec}$ - $\Delta P_{c,\,calc}$), and ($\Delta P_{t,\,spec}$ - $\Delta P_{c,\,calc}$) to zero by changing the cells containing $A_{fr,c}$, N_m, and N_L. Doing so results in

- N_L = 3.08
- N_m = 3.72
- $A_{fr,\,c}$ = 3.24 ft^2 (with N_T = 8 and L_{tube} = 3.24 ft)

Clearly, N_L and N_m must be whole numbers. If they are rounded to N_L = 3 and N_m = 4 (while keeping N_T = 8), and L_{tube} remains 3.24 ft, the results are

- q_{calc} = 2.20E+05 Btu/hr (q_{spec} = 2.25E+05 Btu/hr)
- $\Delta P_{c,\,calc}$ = 0.68 in. WG ($\Delta P_{c,\,spec}$ = 0.7 in. WG)
- $\Delta P_{t,\,calc}$ = 3.3 psi ($\Delta P_{t,\,spec}$ = 4.0 psi)

Now, both pressure drop constraints have been satisfied, but the heat transfer rate is a little low. One could use solver to vary only L_{tube} while driving (q_{spec} - q_{calc}) to zero with N_L = 3, N_m = 4, and N_T = 8. The final result is

- L_{tube} = 3.42 ft for N_T = 8, N_L = 3, N_m = 4
- q_{calc} = 2.25E+05 Btu/hr (q_{spec} = 2.25E+05 Btu/hr)
- $\Delta P_{c,\,calc}$ = 0.62 in. WG ($\Delta P_{c,\,spec}$ = 0.7 in. WG)
- $\Delta P_{t,\,calc}$ = 3.3 psi ($\Delta P_{t,\,spec}$ = 4.0 psi)

As suggested earlier, there are many combinations of N_T and L_{tube} that will give the same $A_{fr,c}$. We used N_T = 8 above. Had we used N_T = 12 instead, the result of the process would have been

- L_{tube} = 2.28 ft for N_T = 12, N_L = 3, N_m = 4
- q_{calc} = 2.25E+05 Btu/hr (q_{spec} = 2.25E+05 Btu/hr)
- $\Delta P_{c,\,calc}$ = 0.62 in. WG ($\Delta P_{c,\,spec}$ = 0.7 in. WG)
- $\Delta P_{t,\,calc}$ = 3.8 psi ($\Delta P_{t,\,spec}$ = 4.0 psi)

The only difference between the two configurations is the higher $\Delta P_{t,\,calc}$ = 3.8 psi for N_T = 12 compared to $\Delta P_{t,\,calc}$ = 3.3 psi for N_T = 8. The difference is a result of the additional U-tubes required in the N_T = 12 configuration. There are 9 passes with 8 U-tubes in the N_T = 12 configuration and 6 passes with 5 U-tubes with N_T = 8. Both configurations satisfy the specifications.

Results and Conclusions

The purpose of this paper was to demonstrate the utility of spreadsheets and their solver capability in heat exchanger rating and design. Using a spreadsheet allows one to easily vary heat exchanger dimensions and see their effect on performance. Using the spreadsheet's nonlinear multivariable solver allows one to easily convert the rating spreadsheet to a design method.

Engineering students become comfortable with spreadsheets early in their academic experience. At the University of Arkansas, spreadsheets are used extensively to solve problems in all thermal science classes. For example, in MEEG 4483 - Thermal Systems Analysis and Design, students are pleased to find that spreadsheets can also be used as the preferred computational tool in heat exchanger rating and design.

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