

# SOLVING PARTIAL DIFFERENTIAL EQUATION WITH MICROSOFT EXCEL TO PREDICT TEMPERATURE PROFILE OF STORED GRAIN

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## Abstract

Microsoft Excel is a powerful tool that can substitute high-level programs to solve complex problems. The undergraduate and graduate students can use the Microsoft Excel spreadsheet and its macro capabilities to solve many complex problems without writing codes in any high-level programming language. This paper describes a method of solving partial differential equation for heat diffusion problems using the Excel spreadsheet. Temperature profile of grains stored for a period of two and a half years was used in developing this method. A one dimensional heat diffusion equation was transformed into finite difference form and used the Microsoft Excel to predict the temperature distribution of the grain mass in an upright storage bin with respect to the variation of the outside air temperature. Certain Excel cells were populated with inputs and parameters related to the finite difference equations. Some macros were written to fit the iteration requirements for the solution. The solution agreed well to the measured grain temperatures. The built-in graphics utility of Excel was used to produce an instantaneous visual effect to the computed result.

Keywords: Microsoft Excel, finite element method, temperature, storage, grain

## Introduction

Commercial software or custom programs are usually used in studying complex problems. Commercial programs are versatile in nature; however still they may have limitations to certain conditions of problems. Custom program requires adequate programming skills of the researchers. Students who are not proficient in writing their own codes, may

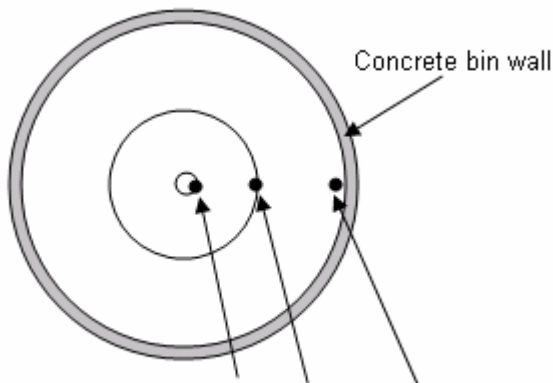
consider Microsoft Excel spreadsheet as an alternative to suit the needs of solving diversified problems. Ketkar and Reddy[3] used Microsoft Excel for numerical solution of an unsteady state heat equation. This paper describes the use of Microsoft Excel in solving a heat diffusion problem with varying boundary conditions

Large grain storage bins are usually constructed with concrete materials. A concrete structure is deterrent to insect infestation and heat diffusion. However, heat diffusion into and out of bin depends on the temperature differentials between either sides of the bin wall. Temperature is the most crucial factor limiting the distribution and abundance of insects, mites and fungi that contaminate and destroy stored grains (Yaciuk, et al, [6]). Seasonal variation of outside air temperature changes the temperature profile throughout a grain bin. Grains are usually stored at safe moisture content. However, a change in grain temperature may cause its moisture content change beyond its safe level initiating qualitative and quantitative damage.

The grains towards the periphery of a bin respond more quickly to the outside air temperature variation than the grains towards the center of the bin. During summer months the temperature around the bin increases. The increase of grain temperature towards the center of the bin lags behind the external increase of temperature. Similarly as the external temperature drops during the winter months, the drops of grain temperatures lag behind the external temperature drops.

This paper uses the data used by Bala et al [1] on wheat stored for two years and a half in a concrete bin of 778.7 m<sup>3</sup> capacity with a

diameter of 5.5m and a height of 33.5m located in Cheney, Kansas. Thermocouples were placed at different locations along the radial direction to measure grain temperature for the duration of storage. A horizontal cross-section of the bin is shown in Figure 1. Bala et al[1] wrote computer programs on both the Finite Element and Finite Different Methods to simulate grain temperatures. But nobody used a simple spreadsheet calculation method to simulate transient problem.



Thermocouples at radii 0.19, 1.39, and 2.61m

Figure 1 Horizontal cross-section of the bin with three thermocouple locations (Figure not drawn to scale)

To develop a one dimensional finite difference model, the following assumptions were made:

- The heat flow pattern was assumed to be symmetric around the vertical axis of the bin
- There was no heat flow in the vertical direction
- The physical and thermal properties of grain was uniform throughout the bin
- There was no heat generation within the grain mass

### Finite Difference Model for Heat Flow

The following partial differential equation describes transient heat transfer in the radial direction.

$$\frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (1)$$

where

T = temperature, °C

t = storage time, hour

r = radial distance from the center of bin, m

α = thermal diffusivities of grain, bin wall materials, and air, m<sup>2</sup>/hour

Analytical solution of this equation is difficult due to the fact that the diffusivity of the three materials are not equal and that the measured boundary condition is not a continuous function of time. The finite difference formulation of equation 1, embedding the Crank-Nicholson's implicit technique[2] is shown in equations (2)

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = \frac{\alpha}{2} \left( \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta r^2} + \frac{1}{i \Delta r} \frac{T_{i+1,j} - T_{i-1,j}}{2 \Delta r} + \frac{T_{i+1,j+1} - 2T_{i,j+1} + T_{i-1,j+1}}{\Delta r^2} + \frac{1}{i \Delta r} \frac{T_{i+1,j+1} - T_{i-1,j+1}}{2 \Delta r} \right) \quad (2)$$

where

Δr = discrete distance along the radial direction of the bin, m

Δt = length of time (time step) between times j and (j+1), h

i = node identity, 0 < i < n (Figure 2)

T<sub>i</sub> = known or estimated temperature at time j

T<sub>j+1</sub> = temperature to be predicted at time, j+1

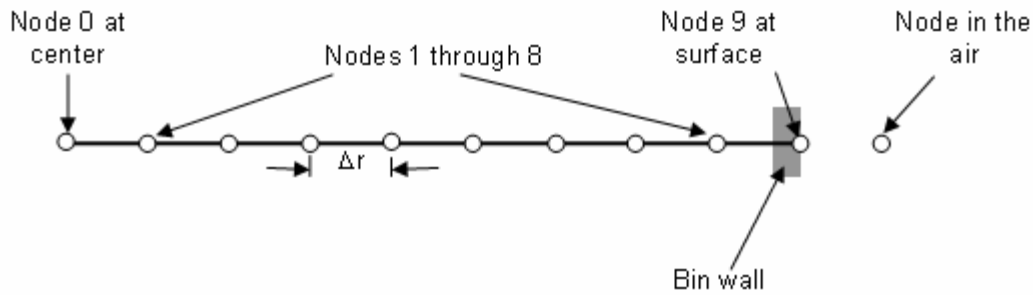


Figure 2 Discrete nodes along radial direction of the bin.

Equation (2) can be rearranged for the known and unknown temperatures as shown below

$$\begin{aligned} & \frac{\alpha \Delta t}{2 \Delta r^2} \left( -1 + \frac{1}{2i} \right) T_{i-1,j+1} + \left( 1 + \frac{\alpha \Delta t}{\Delta r^2} \right) T_{i,j+1} - \frac{\alpha \Delta t}{2 \Delta r^2} \left( 1 + \frac{1}{2i} \right) T_{i+1,j+1} \\ & = \frac{\alpha \Delta t}{2 \Delta r^2} \left( 1 - \frac{1}{2i} \right) T_{i-1,j} + \left( 1 - \frac{\alpha \Delta t}{\Delta r^2} \right) T_{i,j} + \frac{\alpha \Delta t}{2 \Delta r^2} \left( 1 + \frac{1}{2i} \right) T_{i+1,j} \end{aligned} \quad (3)$$

Equation (3) is valid for predicting nodal temperatures for nodes 1 through (n-2) covering only the grains. At node 0, equation (2) is indeterminate because the radius is zero. Nodes (n-1) and n involve the diffusivities of the bin materials and of the air. Individually modified equations for these nodes are given below.

Equation for node 0 is

$$\left( 1 + \frac{2\alpha\Delta t}{\Delta r^2} \right) T_{0,j+1} - \frac{2\alpha\Delta t}{\Delta r^2} T_{1,j+1} = \left( 1 - \frac{2\alpha\Delta t}{\Delta r^2} \right) T_{0,j} + \frac{2\alpha\Delta t}{\Delta r^2} T_{1,j} \quad (4)$$

Equation for node (n-1) is

$$\begin{aligned} & \frac{\Delta t}{2 \Delta r^2} \left( -\alpha_2 + \frac{\alpha_3}{2(n-1)} \right) T_{n-2,j+1} + \left\{ 1 + \frac{\Delta t}{2 \Delta r^2} (\alpha_1 + \alpha_2) \right\} T_{n-1,j+1} - \frac{\Delta t}{2 \Delta r^2} \left( \alpha_1 + \frac{\alpha_3}{2(n-1)} \right) T_{n,j+1} \\ & = \frac{\Delta t}{2 \Delta r^2} \left( \alpha_2 - \frac{\alpha_3}{2(n-1)} \right) T_{n-2,j} + \left\{ 1 - \frac{\Delta t}{2 \Delta r^2} (\alpha_1 + \alpha_2) \right\} T_{n-1,j} + \frac{\Delta t}{2 \Delta r^2} \left( \alpha_1 + \frac{\alpha_3}{2(n-1)} \right) T_{n,j} \end{aligned} \quad (5)$$

Where  $\alpha_1$  = diffusivity for the materials in between the node (n-1) and node n

$\alpha_2 = \alpha$  = diffusivity of grain

$\alpha_3$  = diffusivity for materials in between the node (n-2) and node n

Equation for node n is

$$\begin{aligned} & -\frac{\Delta t}{2 \Delta r^2} (\alpha_{11} + \alpha_{22}) T_{n-1,j+1} + \left\{ 1 + \frac{\Delta t}{2 \Delta r^2} \left( \alpha_1 \left( \frac{2 \Delta r h_c}{k} + 1 \right) + \alpha_{22} + \frac{\alpha_{33}}{2n} \frac{2 \Delta r h_c}{k} \right) \right\} T_{n,j+1} \\ & = \frac{\Delta t}{2 \Delta r^2} (\alpha_{11} + \alpha_{22}) T_{n-1,j} + \left\{ 1 + \frac{\Delta t}{2 \Delta r^2} \left( -\alpha_1 \left( \frac{2 \Delta r h_c}{k} + 1 \right) - \alpha_{22} - \frac{\alpha_{33}}{2n} \frac{2 \Delta r h_c}{k} \right) \right\} T_{n,j} \\ & \quad + \frac{\Delta t h_c}{\Delta r k} \left( \alpha_{11} + \frac{\alpha_{33}}{2n} \right) (T_{a,j} + T_{a,j+1}) \end{aligned} \quad (6)$$

where  $\alpha_{11}$  = diffusivity of air

$\alpha_{22}$  = diffusivity of materials in between node (n-1) and node n

$\alpha_{33}$  = diffusivity of materials in between node (n-1) and a node in air away from node n

## Finite Difference Nodes

In this study the radial dimension of the bin was discretized into 9 equal sections with 10 nodes. These nodes, as mentioned in equations (3), (4), (5) and (6), are shown in Figure 2.

According to the principle of the finite difference method, the temperature at the next step time (at time  $j+1$ ) at node  $i$  will be estimated based on the current temperatures at nodes  $(i-1)$ ,  $i$ , and at  $(i+1)$ . Figure 3 explains this scheme.

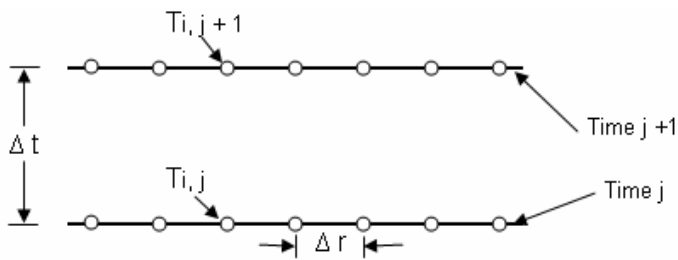


Figure 3 Nodal temperatures at two time spaces.

## Excel Spreadsheet Implementation

The finite difference method yields 10 equations for 10 nodes to find 10 unknown temperatures at time  $t + \Delta t$ . This forms a matrix equation of the following format:

$$AT = H$$

where

A = tri-diagonal constant matrix as function of the thermal diffusion properties of grain, bin wall materials and outside air,

T = vector of unknown nodal temperatures at time,  $t + \Delta t$  (i.e. time,  $j+1$ )

H = known vector as function of the thermal diffusion of grain, bin wall materials and outside air temperatures and known nodal temperatures at time,  $t$  (i.e. time  $j$ )

Matrix A is generated from the constant coefficients of nodal temperatures at time  $j+1$  from equations (3)-(6). Vector H is computed each time from the known values at time  $j$ .

## Equations in the Spreadsheet

Entries in matrix A and in vector H are functions of diffusivities of grain, bin wall materials and of air and nodal distances from the center. Physical and thermal properties as shown in Table 1 are entered in the spreadsheet. Formulas for diffusivities of the three materials are entered. Also entered are the nodal distance ( $\Delta r$ ) and the amount of time step ( $\Delta t$ ).

Table 1. Physical and thermal properties of wheat and concrete.

Properties	Wheat*	Concrete**
Density, $\text{kg.m}^{-3}$	863.0	1950.0
Specific heat, $\text{kJ.kg}^{-1}.\text{K}^{-1}$	1.757	0.653
Thermal conductivity, $\text{W.m}^{-1}\text{K}^{-1}$	0.572	1.082

\* Yaciuk et al[6]

\*\*Mohsenin[4]

Figure 4 shows matrix A and vector H on a Microsoft Excel spreadsheet. The three coefficients at time  $j$  are shown in three columns right before the column for vector H. The spreadsheet multiplies these coefficients with corresponding known temperatures at nodes including the air temperatures for the surface equation to determine the individual entries of vector H.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1																	
2	Constant coefficients at time j+1											Coefficients at time j			Vector H		
3																	
4	Center - Node 0	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Node 9			i-1	i	i+1		
5																	
6																	
7																	
8		b1	c1											f1	g1		h1
9		a2	b2	c2										e2	f2	g2	h2
10			a3	b3	c3									e3	f3	g3	h3
11				a4	b4	c4								e4	f4	g4	h4
12					a5	b5	c5							e5	f5	g5	h5
13						a6	b6	c6						e6	f6	g6	h6
14							a7	b7	c7					e7	f7	g7	h7
15								a8	b8	c8				e8	f8	g8	h8
16									a9	b9	c9			e9	f9	g9	h9
17										a10	b10			e10	f10	g10	h10

Figure 4 The constant tri-diagonal matrix and the known vector H derived from the thermal diffusion properties of gain, bin wall materials and air.

Equation (4) for the node at the center of the bin has two coefficients each at time j and time j+1. Therefore, the first row of matrix A has two entries b1 and c1. Likewise vector H has two entries fi and gi. As for examples, the following coefficients are extracted from equation (4).

$$b1 = 1 + \frac{2\alpha\Delta t}{\Delta r^2}, \quad c1 = -\frac{2\alpha\Delta t}{\Delta r^2},$$

$$f1 = 1 - \frac{2\alpha\Delta t}{\Delta r^2}, \quad g1 = \frac{2\alpha\Delta t}{\Delta r^2}$$

For each of the intermediate nodes as seen in Equation (3), there are three entries, ak, bk and ck for matrix A and three entries, ek, fk, and gk for vector H. The Equation (6) for node n has two coefficients at time j+1. Therefore, the last row of matrix A has two entries. However, the vector H has three entries because the third one comes from the coefficients of ambient air temperatures at time j and at time j+1.

Appropriate equations were written for each entry in matrix A and for each entry in vector H. Matrix A is a tri-diagonal matrix as shown in Figure 4. Solution for the unknown temperatures can be found efficiently using the method described by Thomas[5].

The boundary air temperatures were measured at an interval of 15 days (360 hours). Time step, Δt, was considered as 1, 12, 24, 36, 72, and 360 hours. Any temperature in between two measured temperatures was calculated by straight-line interpolation method. Iterative computation was done using the macro capability of the Excel application.

### Results and Discussion

The measured and the predicted grain temperatures were found to have very good agreement during the total storage period of 780 days. These are exhibited in Figures 5, 6 and 7 at distances 2.61, 1.39 and at 0.18m from the center of the bin respectively. Simulated results were found to be stable for all time steps of 1, 12, 24, 36, 72 and 360 hours. These Figures are based on a time step of 360 hours.

The time difference between (j+1) and j was considered as the time step ( $\Delta t$ ). Prediction at time (j+1) was based on the prediction made time j. A smaller time step is supposed to yield a better prediction. However, the variation of predicted temperatures for  $\Delta t = 360$  hours and that for  $\Delta t = 1$  hour was found to be very minimal. At  $\Delta t = 1$  hour, however, the bin surface temperatures agreed exactly with the ambient air temperatures. This shows the degree of accuracy of this simulation.

The use of  $\Delta t = 360$  hours as time step requires a total of only 53 predictions for the total period of storage. The use of  $\Delta t = 1$  hour as time step requires 360 intermediate predictions for every 15 days resulting in a total of 18,720 predictions for the same period of storage. This extra fineness ( $\Delta t = 1$  hour) does not produce any appreciable improvement in the simulation result except the exact air temperature at the surface of the bin. In other words, the time step of  $\Delta t = 360$  hours is good enough to obtain the desired prediction result. The variation between the predicted and measured temperatures may be attributed to other factors, such as, assumptions used in this study, possible errors in measurement of temperatures, uncounted external temperature variations between two

successive measurement at an interval of 15 days, etc.

Daily variations of outside temperature cause changes in the stored grain with some time delay effect. The grain temperature cycle lags behind the outside air temperature cycle. The lag time increases towards the center of the bin.

Figure 5 shows temperature profile of grains close to the bin wall. Here in this Figure the grain temperature follows very closely to the outside temperature profile throughout the storage period. Also noticed is that the predicted temperature agrees better with the outside air temperature than the measured temperature.

Figure 6 shows temperature profile at a radius of 2.61m from the center of the bin while Figure 7 shows the same near the center of the bin.

The delay of response of grain temperature to outside temperature is widely explicit in Figure 7. Interestingly, by the time grain attained the peak temperature, the air temperature reached to its bottom in its cycle. In other words, the effect of high outside air temperature in summer caused a rise in the grain temperature at the

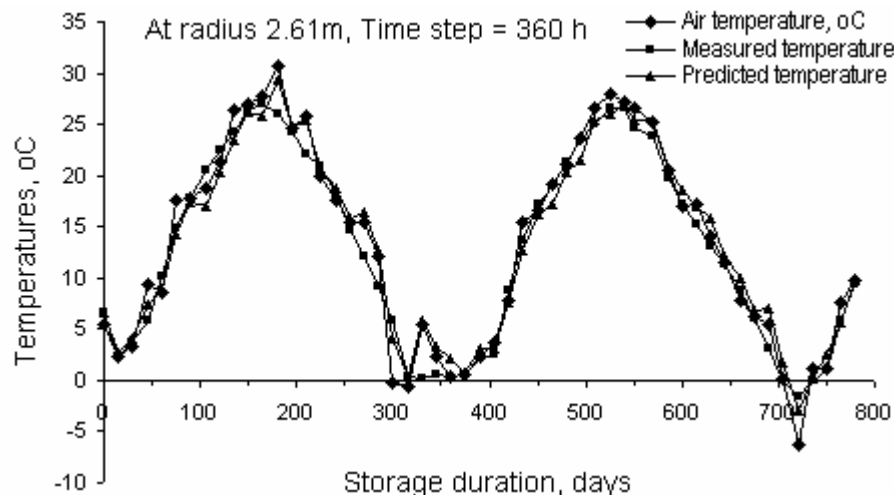


Figure 5 Grain temperature responses to the outside air temperature variations at a point close to the bin wall.

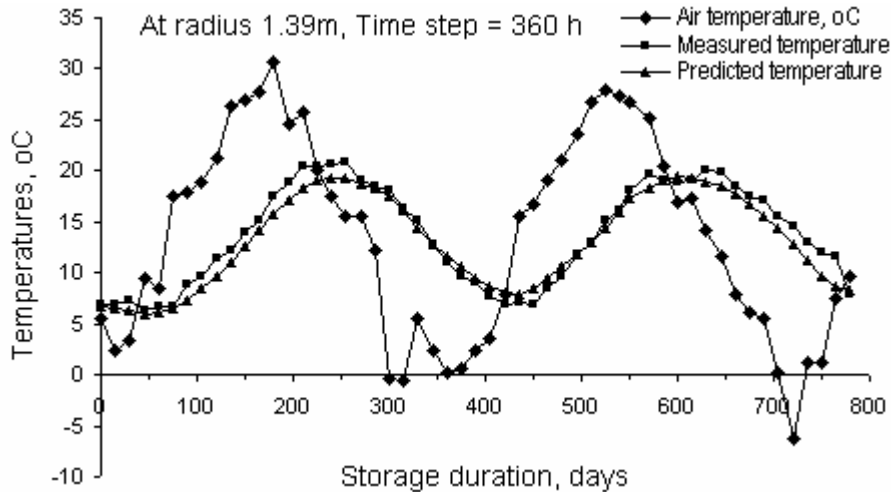


Figure 6 Grain temperature responses to the outside air temperature variations at 1.39m from the center of the bin wall.

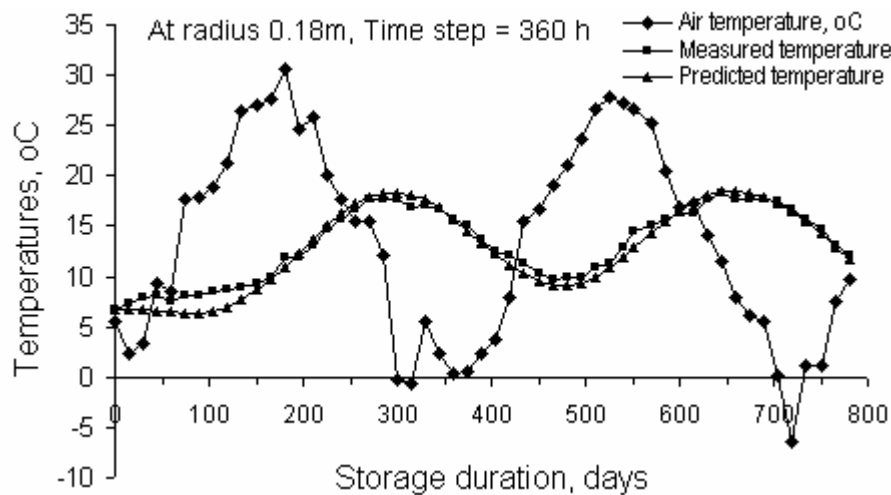


Figure 7 Grain temperature responses to the outside air temperature variations near the bin center.

central area of the bin during the following winter months.

Figure 8 shows the amount of time the grain peak temperatures delayed behind the peak air temperatures. The peak grain temperature at the center was 120 days behind the peak outside air temperature in the first cycle. During the second cycle of air temperature variation, the grain required more time (135 days) to rise to its peak temperature compared to the first cycle. The reason is attributed to the fact that in the

second cycle the air peak rose to about 4 degrees less than that in the first cycle.

### Conclusions

This paper described the development of the solution of a finite difference equation and its implementation using the Microsoft Excel. The predicted results were found to agree pretty well with the measured temperatures throughout the period of storage. Finer time steps, as low as 1 hour, predicted grain temperature better. However, a time steps as high as 360 hours

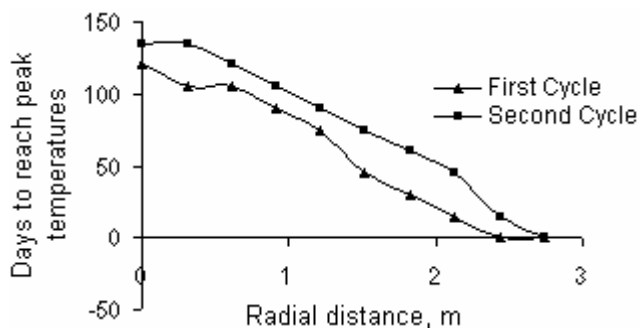


Figure 8 Time lag for grains to reach the peak temperatures following the peak outside air temperatures.

also produced convergent and stable solution and the result is equally acceptable.

This work considered a fixed radial step ( $\Delta r = 0.31\text{m}$ ). A change in the radial step changes the number of nodes and hence the number of equations. Thus, for each variation of the radial step size, a new matrix A and a new vector H are to be formulated in the spreadsheet. The implementation of this approach using the spreadsheet may be cumbersome. Microsoft Excel macros can be used instead. This would increase the burden on programming and would reduce the dependency on the spreadsheet. With some innovative efforts, students can easily develop macros using the readily available help from the Microsoft Excel application.

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