THE SPEED OF SOUND IN STEAM

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Introduction

In their junior year at Webb Institute, students learn about compressible flow. Instruction begins with coverage of isentropic flow of an ideal gas but quickly progresses to the isentropic flow of steam. Ultimately, students learn to do the conceptual design of a steam turbine. Steam is a bit difficult to deal with, because the speed of sound in this medium is a function of both temperature and pressure. There are Mathcad add-in functions that calculate many properties of steam, but the add-in package used at Webb Institute does not include a speed of sound function. For ideal gases, the speed of sound is given by the well know equation

$$c = \sqrt{kRT} \quad [1]$$

where k is the specific heat ratio, R is the specific gas constant, and T is the absolute temperature. However, this equation is not valid for steam.

This paper presents a simple method for finding the speed of sound in steam that is an extension of the existing steam property functions.

The Theory

Early in the derivation of equation [1], the continuity equation and the momentum equation are combined to produce the following

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}. \quad [2].$$

It is important to note that this equation is not restricted to ideal gases. It is this equation that forms the basis for using Mathcad to find the speed of sound in steam.

The set of steam functions for Mathcad includes functions for calculating specific volume and entropy based on pressure and temperature. While it is not possible to take partial derivatives in Mathcad, it is possible to construct a finite difference approximation to such a derivative. Thus

$$c \approx \sqrt{\left(\frac{P_2 - P_1}{\rho_2 - \rho_1}\right)_s} \quad [3]$$

where states 1 and 2 must be fairly close together, and they must have the same entropy.

Implementation in Mathcad

Figure 1 shows the Mathcad worksheet that calculates the speed of sound. The calculations performed by this worksheet are explained below, but users of the worksheet need not concern themselves with these calculations. Working in his or her own Mathcad worksheet, the user need only click Reference from Mathcad's Insert menu, browse to the location of the sound speed worksheet, and select it. Once this is done, the user's own worksheet will calculate the speed of sound in steam from the following syntax

c=sound(P,T). [4]

Behind the scenes, the sound speed worksheet performs the following steps: The first two equations in Figure 1 calculate the entropy and density at the pressure and temperature entered by the user. This is state 1. The third equation increments the pressure by 5 psi, and the fourth equation sets the entropy at the new state equal to the original entropy. Thus state 2 is at the same entropy as state 1 with a pressure 5 psi higher than state 1. The fifth and sixth equations calculate the density and temperature at state 2, and the final equation calculates the speed of sound averaged over the range from state 1 to state 2.

The Mathcad worksheet shown in Figure 1 makes use of several add-in steam property functions. A word about their syntax is in order. All such functions begin with "stm_". The first letter following the underscore is the function's output variable, and the next two letters indicate the input variables. Input variables must be made dimensionless, and the proper units must be attached to the output variable. Extensive use has also been made of Mathcad functions, as distinct from steam property functions. Since the user selects only the temperature and pressure at point 1, the Mathcad functions all have P_1 and T_1 as arguments. This is true even when properties are being calculated at state 2. State 2 is defined in terms of state 1 by incrementing the pressure by 5 psi at constant entropy. For example, the assignment statement

$$s_1(P_1,T_1) := stm\_spt\left(\frac{P_1}{psi}, T_1/^{o}F\right)\frac{BTU}{lbm \cdot R} \qquad [5]$$

calculates the entropy at state 1 from the values of P_1 and T_1. The values of P_1 and T_1 have been made dimensionless, and the units of entropy have been attached. The expression to the left in the assignment statement above is a Mathcad function, while the expression to the right is a steam property function.

Considerable experimentation was involved in finding the appropriate pressure increment. Too small a value gives unreliable results because of the small but finite uncertainty in the specific volume and entropy functions. Too large an increment gives a result that is averaged over too large an interval.

Values calculated by the worksheet shown in Figure 1 were carefully compared with values from the National Institute for Standards and Technology (NIST) Standard Database 12.[1] The NIST values were obtained from the database by means of the REFPROP software. Figures 2 through 5 show comparisons of values for temperatures ranging from 600ºF to 950ºF at four different pressures. Agreement is clearly excellent. The dashed lines, which represent values calculated by Mathcad, are nearly superimposed on the solid lines, which represent values from NIST REFPROP. The standard deviation of all 32 data points is 2.74 feet per second, and the largest difference is 7 feet per second at 1200 psia, 600ºF.

Conclusions

The Mathcad worksheet that calculates the speed of sound in steam is accurate and easy to use. Students at Webb Institute use this worksheet as a tool in doing the conceptual design of a steam turbine, where it is important to ensure that flow remains subsonic.

References

1. NIST Standard Reference Database 12 Version 5.2, 2005.

Biographical Information

Edwin G. Wiggins holds BS, MS, and Ph.D. degrees in chemical, nuclear, and mechanical engineering respectively from Purdue University. He is the Mandell and Lester Rosenblatt Professor of Marine Engineering at Webb Institute in Glen Cove, NY. Ed is a past chairman of the New York Metropolitan Section of the Society of Naval Architects and Marine Engineers (SNAME) and a past regional vice president of SNAME. As a representative of SNAME, Ed Wiggins served on the Technology Accreditation Commission, the Engineering Accreditation Commission, and the Board of Directors of the Accreditation Board for Engineering and Technology (ABET). A Centennial Medallion and a Distinguished Service Award recognize his service to SNAME.

This worksheet contains the functions that calculate the speed of sound in steam when given the temperature and pressure.

$$s_1(P_1,T_1) := \text{stm\_spt}\left(\frac{P_1}{psi}, T_1/°F\right)\frac{BTU}{lbm\cdot R}$$

The entropy at P1, T1

$$\rho_1(P_1,T_1) := \frac{1}{\text{stm\_vpt}\left(\frac{P_1}{psi}, T_1/°F\right)\frac{ft^3}{lbm}}$$

The density at P1, T1

$$P_2(P_1) := P_1 + 5psi$$

Increment the pressure

$$s_2(P_1,T_1) := s_1(P_1,T_1)$$

Keep the entropy the same

$$T_2(P_1,T_1) := \text{stm\_tps}\left(\frac{P_2(P_1)}{psi}, s_2(P_1,T_1)\cdot\frac{lbm\cdot R}{BTU}\right)°F$$

The temperature at state 2

$$\rho_2(P_1,T_1) := \frac{1}{\text{stm\_vpt}\left(\frac{P_2(P_1)}{psi}, T_2(P_1,T_1)/°F\right)\frac{ft^3}{lbm}}$$

The density at state 2

The speed of sound is the square root of the partial derivative of pressure with respect to density at constant entropy. The equation below approximates that calculation.

$$\text{sound}(P_1,T_1) := \sqrt{\frac{P_2(P_1) - P_1}{\rho_2(P_1,T_1) - \rho_1(P_1,T_1)}}$$

Figure 1. Mathcad Worksheet

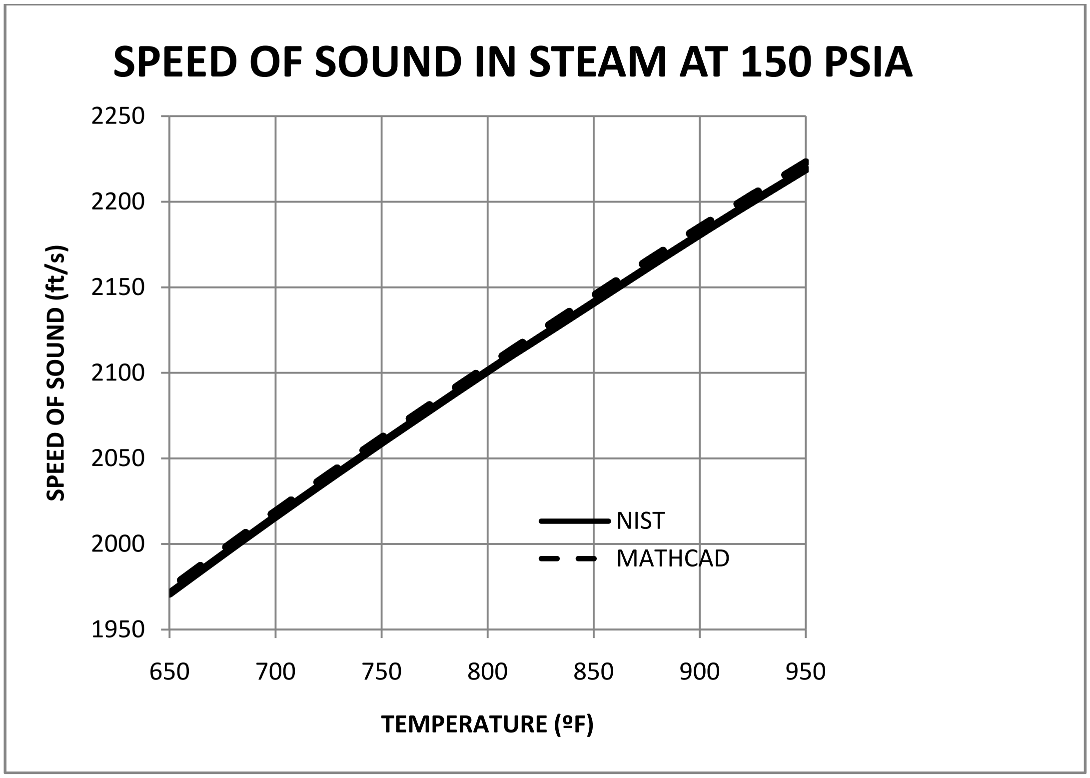

Figure 2. Sound Speed Comparison at 150 psia.

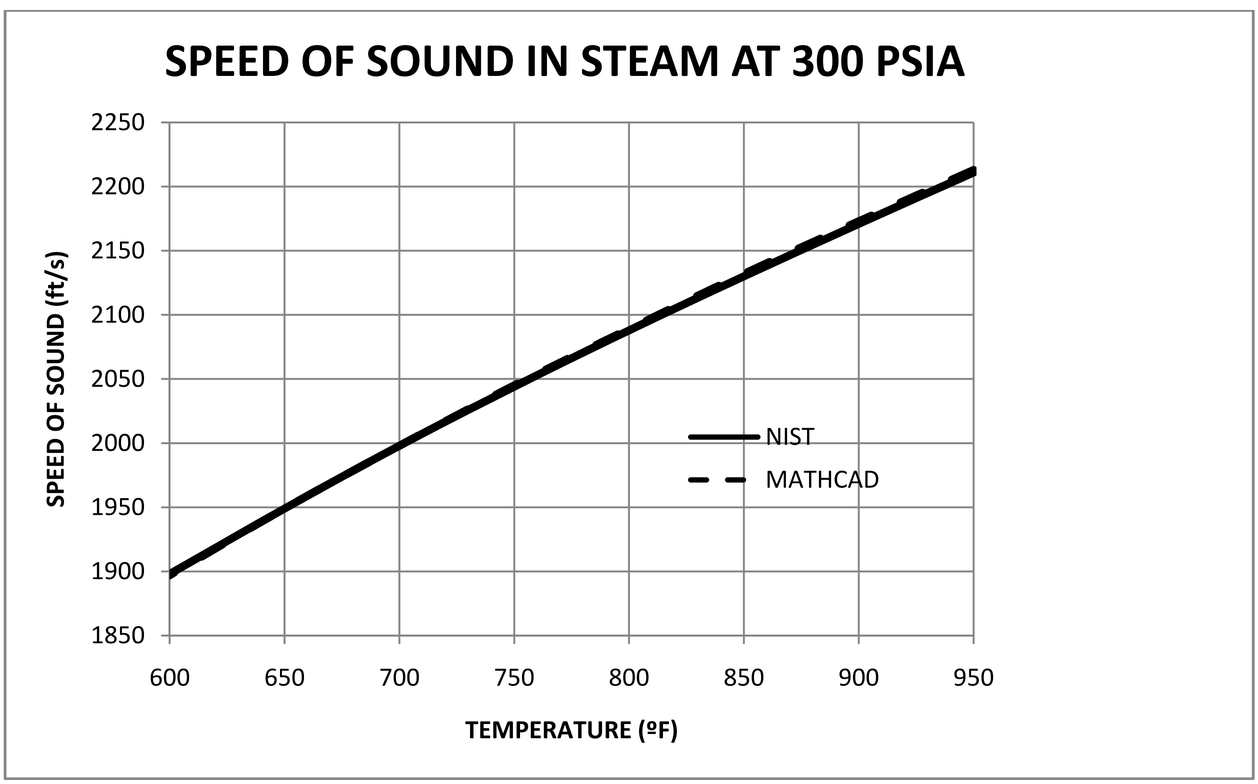

Figure 3. Sound Speed Comparison at 300 psia.

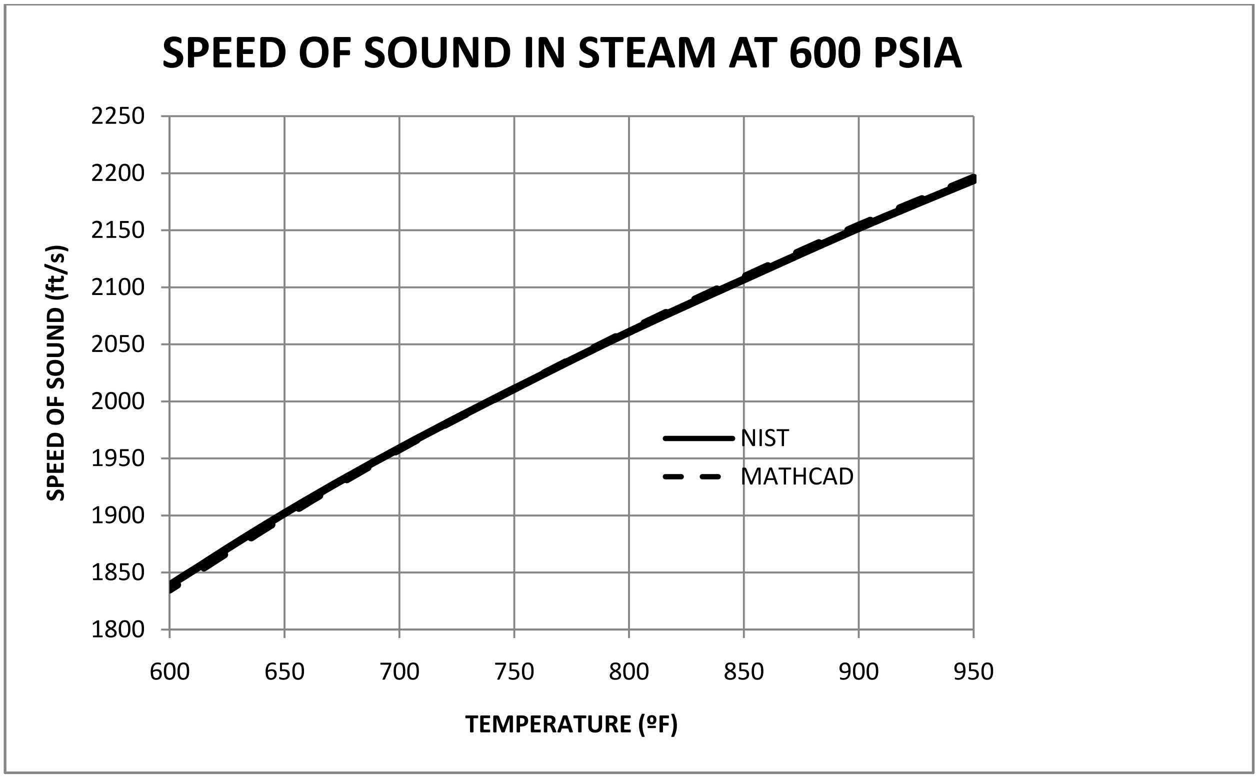

Figure 4. Sound Speed Comparison at 600 psia.

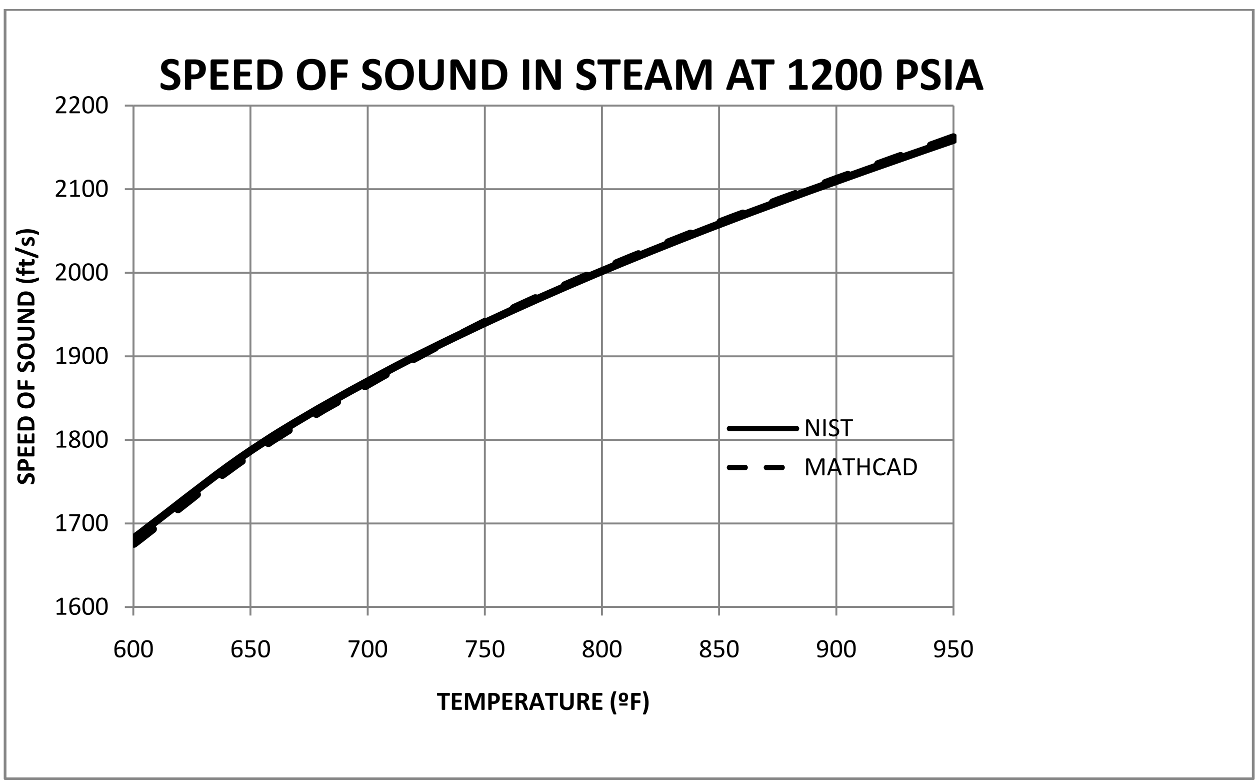

Figure 5. Sound Speed Comparison at 1200 psia.