# ANALOG COMPUTER PROGRAMMING OF HOVERING VTOL DYNAMICS
# PART 1: MATHEMATICAL MODELS

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## Introduction

This work is about the modeling and simulation of some vertical takeoff and landing (VTOL) air-craft in the hover environment. Our goal is to present a reliable and valid approach to putting together generic experimental hardware for VTOL hover research, excluding helicopters and lunar landing vehicles. Many references are included as a useful resource guide.

Seen as a manual or automatic control task, hovering involves the vehicle's staying airborne over a fixed reference point on the ground. Operationally, we define the hovering maneuver as the continuous recovery from small dives, climbs, and turns while tracking a fixed ground reference point to maintain zero error.

## The Assumptions

The six degree-of-freedom (6 DOF) rigid-body equations of motion are written in the body-fixed coordinate system. There are three longitudinal and three lateral/directional motion equations developed by applying Newton's laws. The mathematical models found here and in the literature operate under simplifying assumptions. However, they do describe the significant aspects of the system well. The assumptions are:

1) Both x- and y-axes lie in the plane of symmetry, with the y-axis perpendicular to that plane. The center of gravity (CG) of the aircraft is the origin of a right-hand Cartesian coordinate system.

2) Symmetry is assumed in both the x-y and y-z planes. The three principal axes of inertia coincide with the body-rate torque axes.

3) The airframe is rigid; the geometric orientation between any specified points of the airframe is unalterable.

4) The earth is the inertial reference, a flat plane fixed in space with gravity acting normal to this plane.

5) There is no flow and thus there are neither aerodynamic effects nor aerodynamic stability. There is no relative motion between the earth and its atmosphere.

6) The aircraft's mass is constant. Issues of fuel consumption, distribution, and sloshing are not considered. (Here Newton's law, F = ma, is used in its simplest form with no mass variation in time and a fixed CG.)

7) The gyroscopic effects of rotating power plant components, propellers, or fans are ignored. Note: In the case of specific VTOL aircraft with large rotating masses that generate nontrivial gyroscopic effects, it is wise to rethink the problem.

8) Coupling between the longitudinal and the lateral/directional modes may or may not be considered, depending on desired model complexity (see Burns[5], and this text).

9) Perturbations from equilibrium may or may not follow small-angle assumptions (see this text). We are in favor of retaining nonlinear terms in a mathematical model.

10) Unlike fixed-wing simulations where static stability is an assumed requirement for observing dynamic response, VTOL hover simulations are set up to investigate static stability. Most (but not all) VTOL aircraft are inherently unstable, lacking both attitude and path stability. Many have negative attitude stability. Attitude stability refers to angular displacements (rotational or attitude responses). Path stability refers to translational motion. Static stability is not found in the hovering VTOL aircraft; that is, a disturbance from equilibrium does not introduce restoring forces and moments. Some pendulum- type static stability may exist in a hovering machine with a low CG. Attitude and path stability are discussed.[2,3,4]

11) There are no ground effects due to deflected slipstream, fan, or jet exhaust, etc. Note: Some VTOL power-lift systems introduce nontrivial ground effects which, when known, must be considered in serious simulations of flight in the ground-effect region.

## Developing the Model Equations

The coordinate system nomenclature is:

| Axis | Name | Linear Velocity Along Axis | Angular Velocity Along Axis | Angular Displacements Along Axis | Force Along Axis | Moment | Moment Of Inertia About Axis | Product Of Inertia |
|---|---|---|---|---|---|---|---|---|
| x | Roll | u | p | φ | $F_x$ | L | $I_{xx}$ | $J_{xy}$ |
| y | Pitch | v | q | θ | $F_y$ | M | $I_{yy}$ | $J_{yz}$ |
| z | Yaw | w | r | ψ | $F_z$ | N | $I_{zz}$ | $J_{zx}$ |

First, we write the force equations, relating applied forces $F_{(x,\ y,\ z)}$ to the mass, m, of the vehicle (slugs), to the body-axis angular velocities (rad/sec), and to the body-axis linear velocities (ft/sec):

$$F_x = m\dot{u} + m\left(qw - vr + g\sin\theta\right) \quad (1)$$

$$F_y = m\dot{v} + m\left(ru - pw - g\cos\theta\sin\varphi\right) \quad (2)$$

$$F_z = m\dot{w} + m\left(pv - qu - g\cos\theta\cos\varphi\right) \quad (3)$$

Should the experiment call for small-angle assumptions (because of equipment limitations, etc.), the gravitational forces are mgθ, mgφ, and mg Equations (1), (2), and (3), respectively. In translational form, the above equations are written as:

$$\dot{u} = rv - qw - g\sin\theta \quad (4)$$

$$\dot{v} = pw - ru + g\cos\theta\sin\varphi \quad (5)$$

$$\dot{w} = qu - pv - \left(\frac{T_z}{W} - 1\right) g\cos\theta\cos\varphi \quad (6)$$

where $T_z$ is the thrust along the vertical body axis (up, positive) and W = mg. This format, modified for small-angle assumptions, is found in the hovering aircraft stabilization study[10] and in the earlier work on the use of a 6 DOF motion simulator in VTOL hover research[8] where the additional term $C_z w^2$ appears in Equation (6), $C_z$ denoting a vertical velocity "damping coefficient" (sic) to approximate the X-14A vehicle response.

The torque (or moment) equations relate the moments L, M, and N to the inertias and to the body-axis rates $\dot{p}$, $\dot{q}$, and $\dot{r}$. The equations are developed conveniently in the body-axis system without the added complexity of transforming products and moments of inertia, as is the case for other coordinate systems. It is:

$$L = I_{xx}\,\dot{p} + \left(I_{zz} - I_{yy}\right) q\,r \tag{7}$$

$$M = I_{yy}\,\dot{q} + \left(I_{xx} - I_{zz}\right) r\,p \tag{8}$$

$$N = I_{zz}\,\dot{r} + \left(I_{yy} - I_{xx}\right) p\,q \tag{9}$$

Hovering vehicle simulations use modified forms of Equations (7), (8), and (9) which consider pilot command inputs, control sensitivity terms, and even stabilization feedback terms, as required by specific experiments. To reiterate, in hover the forward velocity is zero and the simulated aircraft has no dynamic stability—aerodynamic derivatives are left out. However, this programming benefit does not exist in transition studies (there are very few of these, see[6]. In rotational form, Equations (7), (8), and (9) are written as:

$$\dot{p} = -\left(\frac{I_{zz} - I_{yy}}{I_{xx}}\right) q\,r \tag{10}$$

$$\dot{q} = -\left(\frac{I_{xx} - I_{zz}}{I_{yy}}\right) r\,p \tag{11}$$

$$\dot{r} = -\left(\frac{I_{yy} - I_{xx}}{I_{zz}}\right) p\,q \tag{12}$$

In piloted simulation we bring in the pilot's control action terms in the form of control deflections ($\delta\varphi$, $\delta\theta$, $\delta\psi$) times the ratio of ($L_{\delta p}$, $M_{\delta q}$, and $N_{\delta r}$) control gains over the ($I_{xx}$, $I_{yy}$, and $I_{zz}$) moments of inertia.

The terms $L_{\delta p}$, $M_{\delta q}$, and $N_{\delta r}$ denote the control gains (note the specialized usage of the word "gain") for roll, pitch, and yaw as they represent the rolling, pitching, and yawing moments per unit of controller deflection (lb-ft/in). The ratios $L_{\delta p}/I_{xx}$, $M_{\delta q}/I_{yy}$, and $N_{\delta r}/I_{ZZ}$ are the respective control sensitivities (rad/sec$^2$/in of controller deflection). Then, the rotational equations for the generic unstabilized VTOL are:

$$\dot{p} = \delta\varphi\left(\frac{L_{\delta p}}{I_{xx}}\right) - \left(\frac{I_{zz} - I_{yy}}{I_{xx}}\right) q\,r \tag{13}$$

$$\dot{q} = \delta\theta\left(\frac{M_{\delta q}}{I_{yy}}\right) - \left(\frac{I_{xx} - I_{zz}}{I_{yy}}\right) r\,p \tag{14}$$

$$\dot{r} = \delta\psi\left(\frac{N_{\delta r}}{I_{zz}}\right) - \left(\frac{I_{yy} - I_{xx}}{I_{zz}}\right) p\,q \tag{15}$$

VTOL handling qualities studies on hover usually include some form of vehicle stabilization which must be derived from the powered lift system. Relying on the powered lift system for both stability and control (no aerodynamics) simplifies things, but at the same time sets heavy design burdens and poses questions of system reliability. There is also the issue of pilot expertise as the independent variables. In the early 1960s, the British Hawker-Siddeley P.1127, the predecessor of the well known Harrier, flew without artificial stabilization, only with a small amount of inherent (aerodynamic) rate damping. Could a "regular" pilot have flown it successfully? We doubt it; so did the Hawker-Siddeley people who later incorporated sophisticated rate-damping mechanisms. Pilot behavior in VTOL aircraft is examined by Schweizer (1965) who discusses the German Dornier Do 31 hovering rig and its stabilization systems (a timely document). In the context of the hovering control problem, a pilot is confronted with great disturbances and the consequences of poor controller design decisions, including inaction, under-correction, ballistic overcompensation, and reversed (wrong direction) inputs.

Rotational equations with stability terms are easily found in the literature.[21,15,9,22,23, 8,10,18,19] Typical are the stabilization feedback terms found in Greif et al.[10]equations as follows:

$$\dot{p} = \delta\phi\left(\frac{L_{\delta p}}{I_{xx}}\right) - \left(\frac{I_{zz} - I_{yy}}{I_{xx}}\right) q\,r + \left(\frac{L_p}{I_{xx}}\right) p + \left(\frac{L_\phi}{I_{xx}}\right)\phi \quad (16)$$

$$\dot{q} = \delta\theta\left(\frac{M_{\delta q}}{I_{yy}}\right) - \left(\frac{I_{xx} - I_{zz}}{I_{yy}}\right) r\,p + \left(\frac{M_q}{I_{yy}}\right) q + \left(\frac{M_\theta}{I_{yy}}\right)\theta \quad (17)$$

$$\dot{r} = \delta\psi\left(\frac{N_{\delta r}}{I_{zz}}\right) - \left(\frac{I_{yy} - I_{xx}}{I_{zz}}\right) p\,q + \left(\frac{N_r}{I_{zz}}\right) r \quad (18)$$

From a point of control, $L_p$, $M_q$, and $N_r$ denote the feedback gains for roll, pitch rate, and yaw rate. Their ratios to the respective moments of inertia ($I_{xx}$, $I_{yy}$, and $I_{zz}$) are the damping rates (1/sec) for roll, pitch, and yaw. The notations $L_\varphi$ and $M_\theta$ denote attitude feedback gains for roll and pitch, and the ratios to their respective moments of inertia ($I_{xx}$ and $I_{yy}$ are the roll and pitch attitude feedback signals).

In Equations (16), (17), and (18) the terms ($L_p/I_{xx}$), ($M_q/I_{yy}$), ($N_r/I_{zz}$), ($L_\varphi/I_{xx}$), and ($M_\theta/I_{yy}$) are negative, thus contributing to damping and attitude stability. Also, $L_p$, $M_q$, and $N_r$ are partial derivatives of the rolling, pitching, and yawing moments with respect to the roll, pitch, and yaw rates (lb-ft/rad/sec). We note that the studies[10,18] assume inertial term ratios $I_{xx}:I_{yy}:I_{zz}$ = 1:2:3 to further simplify the analog computer equations in this case.

The notable assumption that product-of-inertia terms are negligible and can be omitted is made in most hovering studies. Otherwise we may rewrite the rotational equations to include the following terms:

$$+\frac{J_{xz}}{I_{xx}}\left(\dot{r} + p\,q\right) \qquad \text{to the } \dot{p} \text{ equation}$$

$$+\frac{J_{xz}}{I_{yy}}\left(r^2 - p^2\right) \qquad \text{to the } \dot{q} \text{ equation}$$

$$+\frac{J_{xz}}{I_{zz}}\left(\dot{p} - q\,r\right) \qquad \text{to the } \dot{r} \text{ equation}$$

where $J_{xz}$ is the product of inertia about the x- and z-axes (slug-ft$^2$). In the Goldberger[9] study involving a very sophisticated simulation of the Ryan XV-5A aircraft, the following terms were discarded: the pq term in both the $\dot{p}$ and $\dot{r}$ equations, ($J_{xz}/I_{yy}$) term in the $\dot{q}$ equation, and ($J_{xz}/I_{zz}$) term in the $\dot{r}$ equation. Also McLean and Naseem[18] neglected the nonlinear cross-coupling terms qr, rp, and pq in the initial study.

### Some Additional Considerations

The following important points must also be considered in modeling the hovering VTOL machine.

1) Thrust resolution, where a power plant thrust is resolved into horizontal and normal vectors, which may be added to Equations (1) and (3), respectively. A fan-in-wing, fan-in-fuselage, or any other configuration with fixed upward-directed thrust will resolve to a forward-directed horizontal vector with nose-down attitude. Readers are referred to the Goldberger[9] detailed study of the Ryan XV-5A and the fundamental NASA report by Maki and Hickey[16]. It is interesting that the XV-5A vehicle model has been simulated with very simplified model dynamics. Cases in point are found in Elkind et al. and Baron et al.[7,1], parts of a long-term AFFDL-sponsored study on optimal control and human-machine systems. The deflected slipstream VTOL aircraft involves complex lift mechanism aerodynamics. An excellent simulation study of the Ryan VZ-3RY deflected slipstream VTOL airplane is by James et al[14]. Thrust force resolution and effects are, of course, unique for each VTOL configuration, each situation calling for system-specific piloting techniques and automatic-control philosophy (other VTOL configurations are discussed later in this paper).

2) It is prudent to consider power system delays. Transient thrust response is a critical issue in VTOL aircraft.[20]

3) Build in the model as many specific features as possible, as in the case of resolved forces and cross-coupling effects due to deflected slipstream lifting and control surfaces, vectorable exhaust ducts, moving vane assemblies, etc. Cross-coupling is a nemesis in aircraft control, especially in the VTOL vehicle.[17] Inertia cross-coupling is normally ignored in hovering studies. Cross-coupling between lateral, longitudinal, and directional motions happens anytime an aircraft is subject to relatively high rates of rotation and angular accelerations which induce inertia reactions due to an aircraft's distributed mass (see [5,17].

4) Once a valid dynamic/mathematical basic model of the relevant VTOL aircraft has been established, experimentation should include not only automatic control feedback and closures and gains, but also stick command reshaping (see,[13,14,22,23], model-referenced and adaptive control mechanisms. It is important to know how the pertinent lift devices, their operating characteristics, and their geometric arrangement influence things. Here, T. Gardner Hill's article and Coles' paper are useful reading.[11,6]

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