Water Distribution System Design Using Mathematics Application Software

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Introduction

A standard part of Civil and Environmental Engineering curriculums is the subject of hydraulic design. One of the topics covered in hydraulics design courses is the design of water distribution systems. While the principles of designing a water distribution system are fairly straightforward, the application of the principles is complicated by the need to simultaneously solve a large number of equations that describe the network being designed.

This implication of this situation is that either:

a) only extremely simple, non real-world applications are developed in class
b) a more complex, realistic network is designed in which a tremendous amount of effort is expended in a purely mathematical exercise of equation solving that detracts from the main objectives of the project
c) a commercial package is used for the project.

A good combination approach is to go through the development of simple networks from first principles and then use a commercial package for a real world system. Examples of commercial packages include Haestad Methods WaterCad[5], MIKE NET[3] and SynerGee Water[1]. The EPA also has a pipe network modeling package called EPANET[4].

Another alternative that is explored in this paper is the use of mathematical application software as the framework for the design process. The mathematics software is capable of providing the solution of the necessary equations while still requiring that the student develop those equations and all other variables and inputs to perform the analysis. In this way the black box effect of commercial software is removed.

Mathematical Application Software

In the 1980's the first commercial mathematics application software began to surface with the likes of Mathematica[9] and Maple[6]. Mathematical application software is designed for engineers and scientists. They can solve problems ranging from simple to complex, from basic equations to complex calculus and differential equations. At the present time the software is geared toward visual interfaces, rather than line by line commands that used to look similar to writing programming code. Packages have come and gone over the last twenty years with the main market players currently being MathCad[7], Mathematica, Maple and MATLAB[8].

It is expected that current graduating engineers will at least be proficient with spreadsheets and word processing software. Spreadsheets in particular have become the engineers most common tool. At Manhattan College all freshmen engineers are introduced not only to word processing and spreadsheets, but also to a programming language (Visual Basic) and mathematical application software (MathCad). MathCad is the application that will be discussed in this paper.

Application – MathCad and Pipe Network Design

The design of pipe networks is based around writing flow balances and headloss balances for the system. Flow balances are written at each node. Headloss balances are written around closed loops and from points of known head, usually reservoirs. Ultimately there exists a system of n equations for n pipes in the system.

Example of a Simple System

To illustrate the methodology of solving pipe networks consider the simple system shown in Figure 1[11] with corresponding data in Tables 1 and 2.

Table 2. Demand data for simple network.

Node	Elevation (ft)	Demand (cfs)
1	320	2
2	330	4
3	310	1
4	300	3

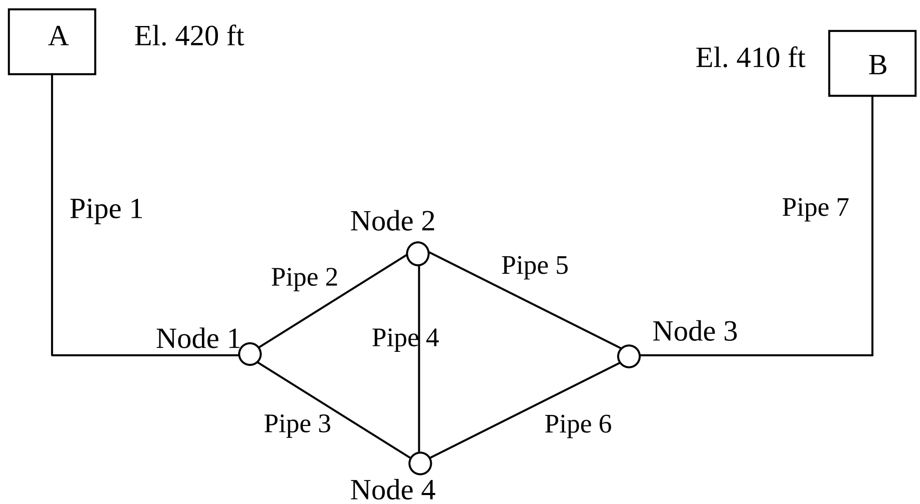

Figure 1. Simple pipe network example.

Table 1. Pipe data for simple network

Pipe	Nodes	Length (ft)	D (in)	f	A (ft^2)	K (s^2/ft^5)
1	A-1	1,000	12	0.015	0.78	0.38
2	1-2	800	8	0.019	0.35	2.89
3	1-4	700	8	0.019	0.35	2.53
4	4-2	750	6	0.020	0.196	12.13
5	3-2	600	8	0.019	0.35	2.17
6	3-4	800	8	0.019	0.35	2.89
7	B-3	900	10	0.017	0.55	0.94

where head loss is given as $h_L = KQ^2$, with $K = fL/DA^2 2g$ from the Darcy-Wiesbach Equation and Q is the flow. f is the pipe friction factor.

This system has 7 pipes and so will require 7 equations. From a flow balance at the four nodes we get:

$$Q1 - Q2 - Q3 = C1$$
$$Q2 + Q4 - Q5 = C2$$
$$-Q5 + Q6 + Q7 = C3$$
$$Q3 - Q4 + Q6 = C4$$

From a headloss balance around the two internal loops we get:

$$K2\,Q2^2 - K4\,Q4^2 - K3\,Q3^2 = 0$$
$$-K5\,Q5^2 + K6\,Q6^2 + K4\,Q4^2 = 0$$

Finally from a headloss balance from Reservoir A to Reservoir B:

$$K1\,Q1^2 + K2\,Q2^2 - K5\,Q5^2 - K7\,Q7^2 = El_A - El_B$$

It is at this point in the problem that the simultaneous solution of the seven equations is needed so that the flows (Q1 to Q7) can be found. A traditional iterative method for achieving this is the Hardy Cross Method [2], which is extremely tedious and predicated by the need to have good initial guesses of the flows. The Hardy Cross method was supplanted in the 1970's by the Linear Method (e.g.[10]). The Linear Method lends itself well for coding into computer programs. Neither of these approaches are particularly appealing in a teaching setting. By contrast the problem can be set out in MathCad exactly as written above and the resulting equations evaluated by the program. The actual MathCad worksheet of the example problem, which was done by students in the class, is shown below.

Example of the Water Distribution System for Eagle Pass, Texas

While the example shown above serves to introduce the methodology, the problems increase in complexity with the introduction of more involved networks and the inclusion of pumping stations. A more complex example from class is the water distribution system for Eagle Pass, Texas[11], as shown below in Figure 2.

This system contains 14 nodes, shown in Figure 2 in circles, at which a demand is exerted by the population served. There are 21 pipes, 5 reservoirs (labeled A – E) at 3 different elevations, and 1 pumping station considered. During class various configurations of this system were analyzed once the basic setup had been completed in MathCad. These included the system with no pumps and with multiple pumps.

The actual MathCad worksheet of a student is shown in Appendix A. The sheet contains all the physical information of the system including elevations, pipe diameters, pipe lengths, node populations, per capita water use, reservoir elevations and the pump characteristic curve. The worksheet shows the solution of the flow through each pipe and from there the calculation of the headloss, the hydraulic grade line (HGL), the pressure head at each node and finally the pressure in psi at each node.

As the software is handling the solution of the equations there is no need to restrict the complexity of problems that are presented and analyzed. Using MathCad, problems analyzed in class have included networks with more than 30 pipes, 6 reservoirs and multiple pumping stations. As evidenced by the worksheets shown here, the methodology is not a black box as students are required to provide all the components of the problem. The software in turn provides a work area and the mathematical solution of the simultaneous equations.

Conclusion

Designing water distribution systems is a core topic in hydraulic analysis courses that all Civil and Environmental Engineering students must take. Modern mathematical application software is well suited to these problems, and allows students to focus on the basic design aspects of the problem rather than the mathematical manipulation required to solve simultaneous equations. An additional benefit of a package like MathCad (the package utilized in the paper) is that the problem is set out exactly as it would be in a notebook or textbook.

Example Network

$El_A := 410ft$ $El_B := 420ft$

$K1 := 0.38\frac{s^2}{ft^5}$ $K2 := 2.89\cdot\frac{s^2}{ft^5}$ $K3 := 2.53\frac{s^2}{ft^5}$ $K4 := 12.13\frac{s^2}{ft^5}$ $K5 := 2.17\frac{s^2}{ft^5}$ $K6 := 2.89\frac{s^2}{ft^5}$ $K7 := 0.94\frac{s^2}{ft^5}$

Initialize Q's

$Q1 := 0\frac{ft^3}{s}$ $Q2 := 0\frac{ft^3}{s}$ $Q3 := 0.\frac{ft^3}{s}$ $Q4 := 0\frac{ft^3}{s}$ $Q5 := 0\frac{ft^3}{s}$ $Q6 := 0\frac{ft^3}{s}$ $Q7 := 0\frac{ft^3}{s}$

Demands

$C1 := 2\frac{ft^3}{s}$ $C2 := 4\frac{ft^3}{s}$ $C3 := 1\frac{ft^3}{s}$ $C4 := 3\frac{ft^3}{s}$

Given

$Q1 - Q2 - Q3 = C1$

$Q2 + Q4 + Q5 = C2$

$-Q5 - Q6 + Q7 = C3$

$Q3 - Q4 + Q6 = C4$

$K2\cdot Q2^2 - K4\cdot Q4^2 - K3\cdot Q3^2 = 0$

$-K5\cdot Q5^2 + K6\cdot Q6^2 + K4\cdot Q4^2 = 0$

$K1\cdot Q1^2 + K2\cdot Q2^2 - K5\cdot Q5^2 - K7\cdot Q7^2 = El_A - El_B$

$Find(Q1, Q2, Q3, Q4, Q5, Q6, Q7) = \blacksquare \frac{ft^3}{s}$

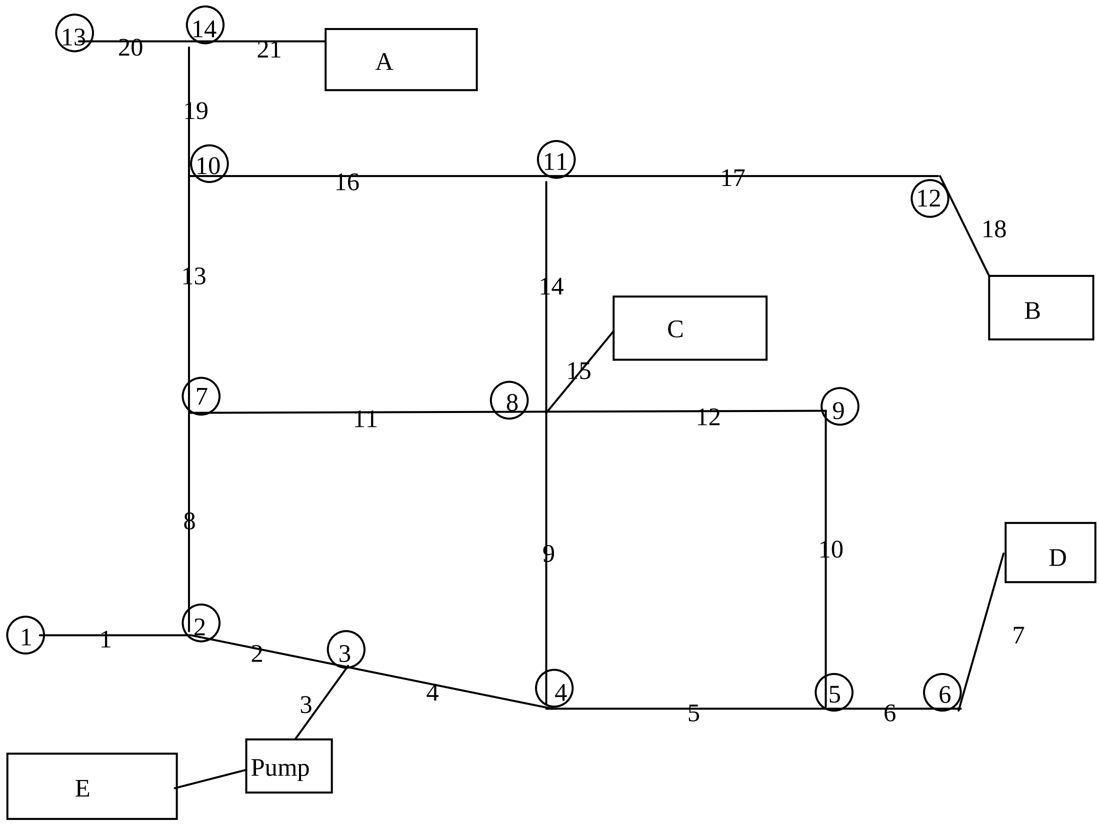

Figure 2. Schematic of water distribution system for Eagle Pass, Texas.

References

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10. Wood, D. J. and Charles, C. O. A. (1972) Hydraulic analysis using linear theory, Journal of Hydraulics Division, ASCE, Vol. 107, No. 10.

11. Wurbs, R.A and James, W. P. (2002) Water Resources Engineering, Prentice Hall, NJ.

Biographical Information

Scott Lowe is an Associate Professor in the Civil and Environmental Engineering Department at Manhattan College in Riverdale, New York. He is also a Senior Project Manager at Lawler, Matusky and Skelly Engineers in Pearl River, New York. He specialized in numerical modeling of environment problems including air quality, water quality and fluid mechanics. He has taught this course on Air Quality Models for the past 10 years.

Appendix A: MathCad Worksheet for Eagle Pass, Texas.

NOTE: Schematic for Eagle Pass, TX provided

Data & Calculations:

Line	Length (ft)	Diameter	D (ft.)	A^2	k
1	1400	12	1.00	0.62	0.71
2	1300	14	1.17	1.14	0.30
3	700	14	1.17	1.14	0.16
4	1500	14	1.17	1.14	0.35
5	2000	12	1.00	0.62	1.01
6	1000	12	1.00	0.62	0.50
7	2500	10	0.83	0.30	3.14
8	2000	12	1.00	0.62	1.01
9	3000	12	1.00	0.62	1.51
10	3000	12	1.00	0.62	1.51
11	2600	12	1.00	0.62	1.31
12	2000	10	0.83	0.30	2.51
13	1100	12	1.00	0.62	0.55
14	1100	10	0.83	0.30	1.38
15	400	10	0.83	0.30	0.50
16	2600	10	0.83	0.30	3.26
17	3400	10	0.83	0.30	4.26
18	1800	10	0.83	0.30	2.26
19	1400	10	0.83	0.30	1.76
20	900	10	0.83	0.30	1.13
21	550	10	0.83	0.30	0.69

Note: A = Area of a Circle

$k = f\,(L/D)\,(1/2g)\,(1/A^2)$

Where f = 0.020

Provided Information:

$EL_A := 950ft$ $EL_B := 950ft$ $EL_C := 880ft$ $EL_D := 880ft$ $EL_E := 770ft$

Define k values for Eagle Pass, TX (as determined in Excel):

$k_1 := 0.71\frac{s^2}{ft^5}$ $k_2 := 0.30\frac{s^2}{ft^5}$ $k_3 := 0.16\frac{s^2}{ft^5}$ $k_4 := 0.35\frac{s^2}{ft^5}$

$k_5 := 1.01\frac{s^2}{ft^5}$ $k_6 := 0.50\frac{s^2}{ft^5}$ $k_7 := 3.14\frac{s^2}{ft^5}$ $k_8 := 1.01\frac{s^2}{ft^5}$

$k_9 := 1.51\frac{s^2}{ft^5}$ $k_{10} := 1.51\frac{s^2}{ft^5}$ $k_{11} := 1.31\frac{s^2}{ft^5}$ $k_{12} := 2.51\frac{s^2}{ft^5}$

$k_{13} := 0.55\frac{s^2}{ft^5}$ $k_{14} := 1.38\frac{s^2}{ft^5}$ $k_{15} := 0.50\frac{s^2}{ft^5}$ $k_{16} := 3.26\frac{s^2}{ft^5}$

$k_{17} := 4.26\frac{s^2}{ft^5}$ $k_{18} := 2.26\frac{s^2}{ft^5}$ $k_{19} := 1.76\frac{s^2}{ft^5}$ $k_{20} := 1.13\frac{s^2}{ft^5}$

$k_{21} := 0.69\frac{s^2}{ft^5}$

Populations

$p1 := 1250$ $p2 := 2000$ $p3 := 1500$ $p4 := 1900$ $p5 := 2250$ $p6 := 2800$ $p7 := 750$

$p8 := 2500$ $p9 := 1000$ $p10 := 500$ $p11 := 1250$ $p12 := 4000$ $p13 := 1800$ $p14 := 1500$

Define values of C (Given):

$$use := 160\frac{gal}{day}$$

$c_1 := p1 \bullet use$ $c_2 := p2 \bullet use$ $c_3 := p3 \bullet use$ $c_4 := p4 \bullet use$ $c_5 := p5 \bullet use$ $c_6 := p6 \bullet use$

$c_7 := p7 \bullet use$ $c_8 := p8 \bullet use$ $c_9 := p9 \bullet use$ $c_{10} := p10 \bullet use$ $c_{11} := p11 \bullet use$

$c_{12} := p12 \bullet use$ $c_{13} := p13 \bullet use$ $c_{14} := p14 \bullet use$

Initial guess values for Q:

$Q_1 := 0\frac{ft^3}{s}$ $Q_2 := 0\frac{ft^3}{s}$ $Q_3 := 0\frac{ft^3}{s}$ $Q_4 := 0\frac{ft^3}{s}$ $Q_5 := 0\frac{ft^3}{s}$

$Q_6 := 0\frac{ft^3}{s}$ $Q_7 := 0\frac{ft^3}{s}$ $Q_8 := 0\frac{ft^3}{s}$ $Q_9 := 0\frac{ft^3}{s}$ $Q_{10} := 0\frac{ft^3}{s}$

$Q_{11} := 0\frac{ft^3}{s}$ $Q_{12} := 0\frac{ft^3}{s}$ $Q_{13} := 0\frac{ft^3}{s}$ $Q_{14} := 0\frac{ft^3}{s}$ $Q_{15} := 0\frac{ft^3}{s}$

$Q_{16} := 0\frac{ft^3}{s}$ $Q_{17} := 0\frac{ft^3}{s}$ $Q_{18} := 0\frac{ft^3}{s}$ $Q_{19} := 0\frac{ft^3}{s}$ $Q_{20} := 0\frac{ft^3}{s}$

$Q_{21} := 0\frac{ft^3}{s}$

Pump Equations (Linear)

$$E_P := 115 \bullet ft - 3.5\frac{s}{ft^2} \bullet Q_3$$

Write down equations for: Flow Balance, Headloss, & Total Headloss for this system:

Given

$$Q_1 - c_1 = 0$$

$$Q_8 - Q_1 - Q_2 - c_2 = 0$$

$$Q_3 + Q_2 - Q_4 - c_3 = 0$$

$$Q_9 + Q_4 + Q_5 - c_4 = 0$$

$$Q_6 - Q_5 + Q_{10} - c_5 = 0$$

$$-Q_6 + Q_7 - c_6 = 0$$

$$-Q_8 + Q_{13} - Q_{11} - c_7 = 0$$

$$Q_{11} + Q_{15} + Q_{14} - Q_{12} - Q_9 - c_8 = 0$$

$$Q_{12} - Q_{10} - c_9 = 0$$

$$Q_{19} - Q_{16} - Q_{13} - c_{10} = 0$$

$$Q_{16} - Q_{14} + Q_{17} - c_{11} = 0$$

$$-Q_{17} + Q_{18} - c_{12} = 0$$

$$Q_{20} - c_{13} = 0$$

$$-Q_{20} + Q_{21} - Q_{19} - c_{14} = 0$$

$$k_{16} \bullet Q_{16}^2 + k_{14} \bullet Q_{14}^2 - k_{11} \bullet Q_{11}^2 - k_{13} \bullet Q_{13}^2 = 0$$

$$k_{11} \bullet Q_{11}^2 + k_9 \bullet Q_9^2 - k_4 \bullet Q_4^2 - k_2 \bullet Q_2^2 - k_8 \bullet Q_8^2 = 0$$

$$k_{12} \bullet Q_{12}^2 + k_{10} \bullet Q_{10}^2 + k_5 \bullet Q_5^2 - k_9 \bullet Q_9^2 = 0$$

$$k_{21} \bullet Q_{21}^2 + k_{19} \bullet Q_{19}^2 + k_{16} \bullet Q_{16}^2 - k_{17} \bullet Q_{17}^2 - k_{18} \bullet Q_{18}^2 = EL_A - EL_B$$

$$k_{15} \bullet Q_{15}^2 + k_9 \bullet Q_9^2 - k_4 \bullet Q_4^2 - k_3 \bullet Q_3^2 = EL_C - EL_E - 115ft + 3.5\frac{s}{ft^2} \bullet Q_3$$

$$k_{15} \bullet Q_{15}^2 + k_9 \bullet Q_9^2 - k_5 \bullet Q_5^2 - k_6 \bullet Q_6^2 - k_7 \bullet Q_7^2 = EL_C - EL_D$$

$$k_7 \bullet Q_7^2 + k_6 \bullet Q_6^2 + k_5 \bullet Q_5^2 - k_4 \bullet Q_4^2 - k_3 \bullet Q_3^2 = EL_D - EL_E - 115ft + 3.5\frac{s}{ft^2} \bullet Q_3$$

$$\begin{pmatrix} Q_{1} \\ Q_{2} \\ Q_{3} \\ Q_{4} \\ Q_{5} \\ Q_{6} \\ Q_{7} \\ Q_{8} \\ Q_{9} \\ Q_{10} \\ Q_{11} \\ Q_{12} \\ Q_{13} \\ Q_{14} \\ Q_{15} \\ Q_{16} \\ Q_{17} \\ Q_{18} \\ Q_{19} \\ Q_{20} \\ Q_{21} \end{pmatrix} = \quad \frac{gal}{min}$$

	0
0	138.889
1	12.945
2	771.542
3	617.82
4	-51.131
5	58.626
6	369.737
7	374.057
8	-355.579
9	140.243
10	-253.291
11	251.354
12	204.099
13	-269.856
14	696.701
15	-44.519
16	-86.449
17	357.996
18	215.136
19	200
20	581.802
21	

Check flow continuity:

$$Demand := c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7 + c_8 + c_9 + c_{10} + c_{11} + c_{12} + c_{13} + c_{14}$$

$$Demand = 0.175\frac{m^3}{s}$$

$$Supply := Q_{21} + Q_{18} + Q_{15} + Q_7 + Q_3$$

$$Supply = 0.175\frac{m^3}{s}$$