USING MATHCAD AND THE AMERICAN INSTITUTE OF STEEL CONSTRUCTION DATABASE TO DESIGN AND ANALYZE WIDE-FLANGE SHAPES UNDER COMBINED AXIAL COMPRESSION AND FLEXURAL LOADING FOR LRFD

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INTRODUCTION

With the third edition of the Manual of Steel Construction for Load and Resistance Factor Design (LRFD), the American Institute of Steel Construction (AISC) has adopted a design method for wide flange shapes (W-shapes) under combined axial and flexural loading that lends itself to solution via computer code. This paper shows one such code using Mathcad and Mathcad's Excel component to design and analyze wide flange steel shapes for any combination of axial compressive and flexural loads. The one page Mathcad worksheet (known as the W-shape Design Wizard, or wizard for short) easily allows instructors or students to input load effects, geometry, support conditions, and steel properties for an individual structural member. The wizard then searches the AISC database of W-shapes for those shapes that have enough combined strength to satisfy the LRFD interaction equation and returns the specified number of shapes and nominal depth in a list from lightest to heaviest. The user then may select any of these adequate shapes from a drop-down list to get a complete analysis of the shape to include its geometric properties, individual strengths (axial and flexural), as well as its strength interaction value.

This worksheet has been used successfully over the last several years by the faculty at the United States Military Academy to quickly and efficiently create design problems, problem sets, and exams as well as to post interactive solutions to web sites and check student work on design problems. This paper describes the design of W-shapes using the LRFD method for combined axial compression and flexure, shows how the code solves the design problem, and then uses the wizard to solve two design problems.

BACKGROUND: DESIGNING WIDE FLANGE SHAPES FOR COMBINED AXIAL COMPRESSION AND FLEXURE USING THE B,M,N METHOD.

AISC Design Requirements

Wide Flange shapes are designed to carry loads in flexure about their strong (x-x) axis (see figure 1) when used as a beam or girder and compressive axial loads when used as columns. However, in common structural applications W-shapes are often subject to combined axial and flexural loads. Additionally, the flexural load may be applied about the W-shape's strong (x-x) or weak (y-y) axis. When any combination of these three conditions occur, the Load and Resistance Factor Design Specification for Structural Steel Buildings requires that "the interaction of flexure and compression in symmetric shapes be limited by equations H1-1a and H1-1b" (AISC, 16.1-38) shown below.

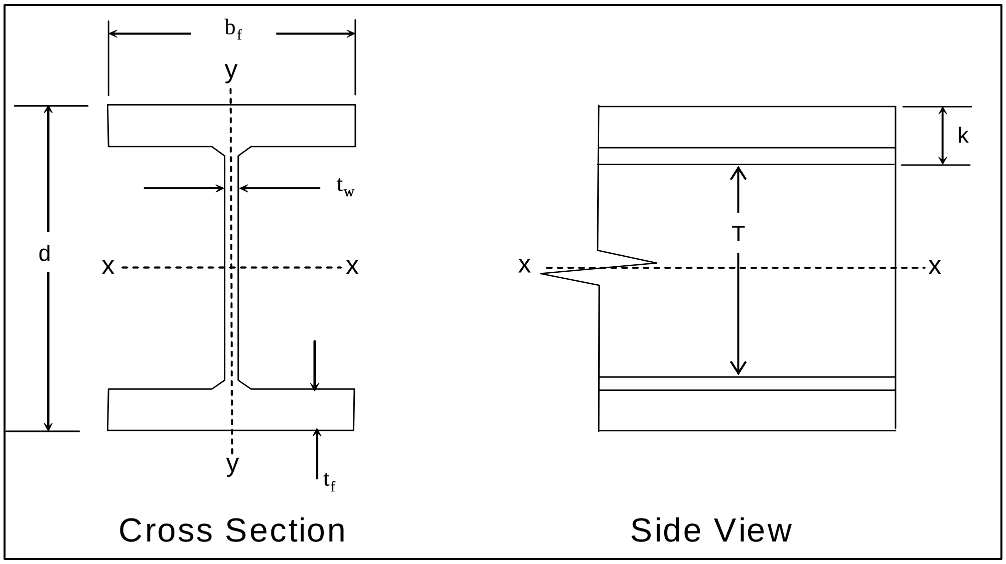

Figure 1 Wide Flange (W) Shape

For $\frac{P_u}{\phi P_n} \geq 0.2$ $\quad \frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) \leq 1.0$ (LRFD EQN H1-1a)

For $\frac{P_u}{\phi P_n} < 0.2$ $\quad \frac{P_u}{2\phi P_n} + \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) \leq 1.0$ (LRFD EQN H1-1b).

where
P_u = required compressive strength, kips
M_{ux} = required flexural strength about x-x axis, kip-ft
M_{uy} = required flexural strength about y-y axis, kip-ft
ϕP_n = compressive design strength, kips
ϕM_{nx} = flexural design strength about x-x axis, kip – ft
ϕMny = flexural design strength about y-y axis, kip - ft

The required strengths listed above (also known as load effects since they are internal member forces that are induced by external loads on the member) can be determined by an appropriate structural analysis method. The design strengths listed above are determined by applying the appropriate LRFD code equations which account for a members material strength and any reduction in strength due to local or global instability (see LRFD specification chapters B, C, E, and F). Once the appropriate load effects and design strengths have been determined, these values are then combined in the appropriate interaction equation (Equations H1-1a or H1-1b) to determine if the interaction value is below one (called a safe condition) or above one (called an unsafe condition). If the interaction value is above one, the member must be redesigned. If the interaction value is equal to or below one, then the member is adequate, but may not be efficient (it may be larger than required). Typically, the member would be redesigned until its interaction value was nearly, but not over one.

b,m,n Design Method

In order to aid in this rather tedious and often iterative design process, AISC has adopted a design method developed by Aminmansour. This method (called here the b,m,n method) tabulates the inverse of the design strengths (axial compression, and flexural about the weak and strong axis) used in equation H1-1a against discrete effective and unbraced lengths for each W-Shape in the AISC database. Equation H1-1a is then rewritten using the variables b, m, and n to represent the appropriate value of the inverse of the axial, flexural (x-x axis), and flexural (y-y axis) strengths respectively giving the equation: $bP_u + mM_{ux} + nM_{uy} \le 1.0$, where $b = \frac{1}{\phi P_n}$, $m = \frac{8}{9\phi M_{nx}}$, and $n = \frac{8}{9\phi M_{ny}}$. The engineer can now solve the interaction inequality by selecting two of three variables (b, m, or n) and solving for the third. Then he or she would look up an appropriate cross section in the design chart based on the solved variable and the appropriate effective or unbraced length. Then the engineer must then analyze the selected cross section to ensure its interaction value is less than one as well as decided if the cross section is efficient. Even with a fairly accurate guess of the initial two variables (AISC provides a table of median values for each nominal W-Shape depth in its Manual of Steel Construction to aid in this effort), this method typically results in several iterations to find an adequate and efficient W-shape. Therefore this process is a good candidate for automation to save time and reduce the likelihood of errors in the design process.

THE W-SHAPE DESIGN AND ANALYSIS WIZARD

Introduction

Figure 2 shows the W-Shape Design and Analysis Wizard. In the top portion of the wizard the user inputs the loads, end conditions,

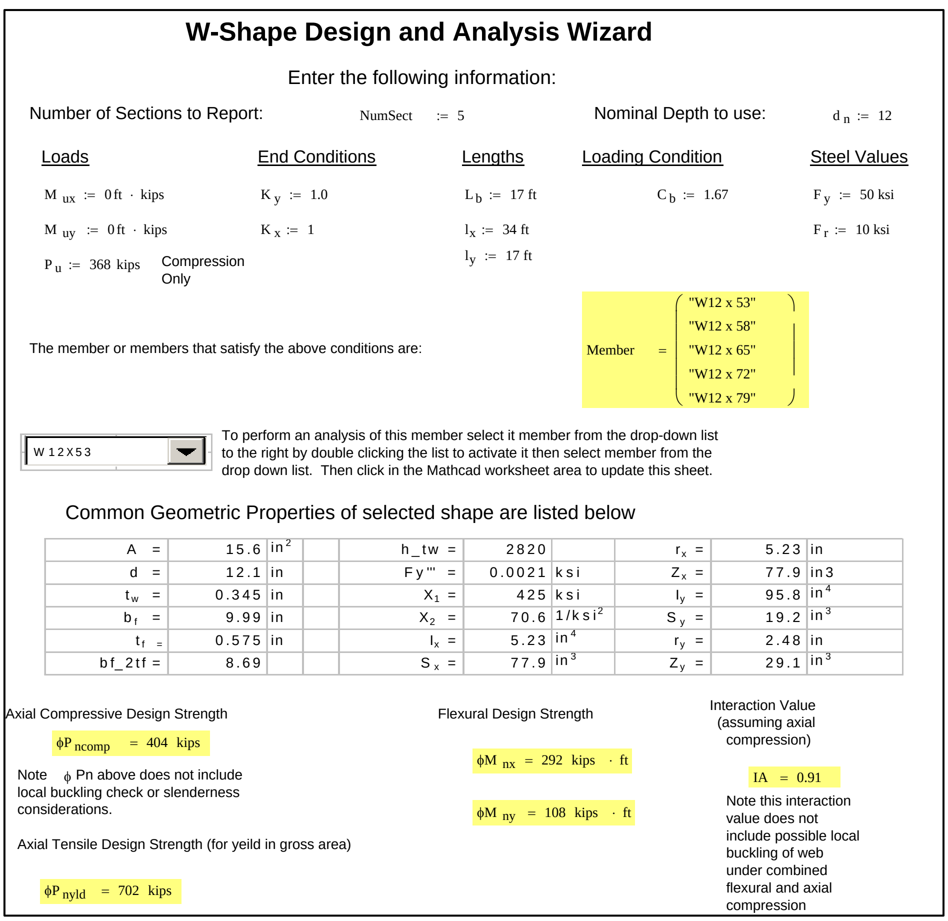

Figure 2 W-Shape Design and Analysis Wizard

lengths, loading condition and steel properties associated with the problem (all variables are standard to the AISC Manual of Steel Construction). The worksheet then returns the W-shapes from the AISC database that will satisfy the given conditions. Then the user may select the desired cross-section from the drop down list of AISC tabulated W-Shapes to analyze it for the specific design conditions.

Selecting and Organizing Acceptable Designed W-Shapes from AISC Database

The code that runs the design portion of the wizard is broken down into several different functions. The main function (called BCd_n) is shown in Figure 3. This function selects the number of sections of the nominal depth specified from the array of weight-ordered adequate shapes. To get the array of weight-ordered adequate shapes organized from lightest to heaviest shape this function must call function BC (shown in Figure 4). Function BC orders the array of acceptable shapes from lightest to heaviest. In order to get the array of acceptable shapes, function BC calls function BC_{index} (shown in Figure 5). Function BC_{index} in turn calls the necessary functions to check the design strengths and interaction values for each W-shape in the AISC database. Function BC_{index} then returns each adequate shape's index number to function BC. The hierarchy of all required functions is shown on the chart in Figure 6. All other functions in the hierarchy are shown in Appendix A.

```
BCd_n(d_n, NumSect, K_x, K_y, l_x, l_y, L_b, C_b, F_y, F_r, P_u, M_ux, M_uy) :=
    BCpossible ← BC(K_x, K_y, l_x, l_y, L_b, C_b, F_y, F_r, P_u, M_ux, M_uy)
    i ← 1
    j ← 1
    while i ≤ NumSect
        if d_n = (BCpossible^<2>)_j
            BCd^<i> ← (BCpossible^T)^<j>
            i ← i + 1
        j ← j + 1
    BCd^T
```

Figure 3 Function BCd_n

```
BC(K_x, K_y, l_x, l_y, L_b, C_b, F_y, F_r, P_u, M_ux, M_uy) :=
    BC_ind ← BC_index(K_x, K_y, l_x, l_y, L_b, C_b, F_y, F_r, P_u, M_ux, M_uy)
    i ← 0
    for j ∈ BC_ind
        i ← i + 1
        BC^<i> ← ((DB)^T)^<j>
    BC ← BC^T
    csort(BC, 3)
```

Figure 4 Function BC

$$BC_{index}(K_x, K_y, l_x, l_y, L_b, C_b, F_y, F_r, P_u, M_{ux}, M_{uy}) :=$$

$$
\begin{array}{l}
j \leftarrow 1 \\
\text{for } i \in 1..\text{rows}(DB) \\
\quad \text{if } \left(DB^{\langle 1 \rangle}\right)_i = 1 \\
\quad\quad b \leftarrow \text{Findb}(F_{cr}(K_x, K_y, l_x, l_y, F_y, i), i) \\
\quad\quad m \leftarrow \text{Findm}\left(M_{ncompact}(C_b, L_b, L_p(F_y, i), L_r(F_y, F_r, i), M_{px}(F_y, i), M_{rx}(F_L(F_y, F_r), i), i), M_{nFLB}\left(M_{px}(F_y, i), M_{rx}(F_L(F_y, F_r), i), F_y, F_L(F_y, F_r), S_{x_i}, i\right)\right) \\
\quad\quad n \leftarrow \text{Findn}\left(M_{py}(F_y, i), M_{nFLB}\left(M_{py}(F_y, i), M_{ry}(F_L(F_y, F_r), i), F_y, F_L(F_y, F_r), S_{y_i}, i\right)\right) \\
\quad\quad \text{if } b \cdot P_u < .2 \wedge \left(.5 \cdot b \cdot P_u + \frac{9}{8} m \cdot M_{ux} + \frac{9}{8} \cdot n \cdot M_{uy}\right) \le 1 \\
\quad\quad\quad index_j \leftarrow i \\
\quad\quad\quad j \leftarrow j + 1 \\
\quad\quad \text{if } b \cdot P_u \ge .2 \wedge \left(b \cdot P_u + m \cdot M_{ux} + n \cdot M_{uy}\right) \le 1 \\
\quad\quad\quad index_j \leftarrow i \\
\quad\quad\quad j \leftarrow j + 1 \\
index
\end{array}
$$

Figure 5 Function BC_{index}

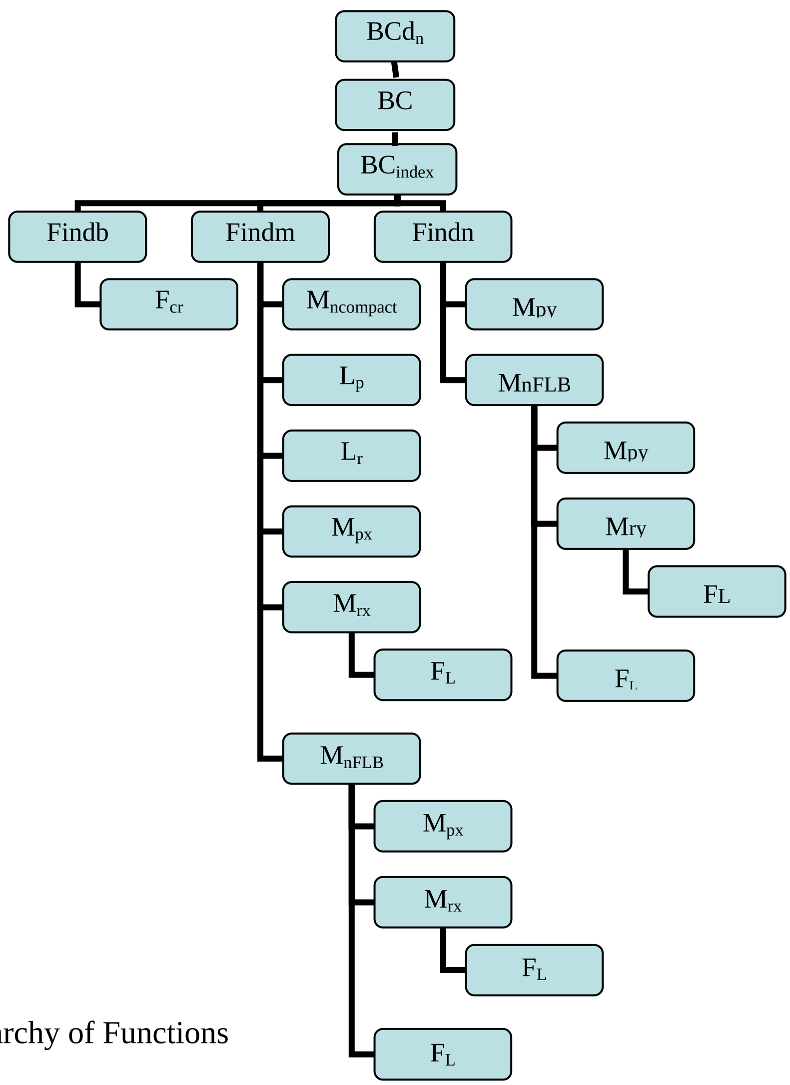

Figure 6 Hierarchy of Functions

Drawing Cross-Sections from the AISC Database

In order to access the AISC database information function BC and BC_{index} use a file named AISC_GetShapesv_3 (Shown in Figure 7). This file reads in the third edition of the AISC database in as an excel object to the Mathcad file in a variable called DB. Then, using Mathcad's Excel component each column of the database (containing the geometric properties of the W-Shapes) is named, and the

$DB :=$ Worksheet

$$d_n := DB^{\langle 2 \rangle} \qquad Wt := DB^{\langle 3 \rangle}\frac{lbf}{ft} \qquad A := DB^{\langle 4 \rangle}\cdot in^2$$

$$d := DB^{\langle 5 \rangle}\cdot in \qquad OD := DB^{\langle 6 \rangle}\cdot in \qquad b_f := DB^{\langle 7 \rangle}\cdot in$$

$$b := DB^{\langle 8 \rangle}\cdot in \qquad ID := DB^{\langle 9 \rangle}\cdot in \qquad t_w := DB^{\langle 10 \rangle}\cdot in$$

$$t_f := DB^{\langle 11 \rangle}\cdot in \qquad t := DB^{\langle 12 \rangle}\cdot in \qquad k := DB^{\langle 13 \rangle}\cdot in$$

$$x := DB^{\langle 14 \rangle}\cdot in \qquad y := DB^{\langle 15 \rangle}\cdot in \qquad e_0 := DB^{\langle 16 \rangle}\cdot in$$

$$x_p := DB^{\langle 17 \rangle}\cdot in \qquad y_p := DB^{\langle 18 \rangle}\cdot in \qquad bf\_2tf := DB^{\langle 19 \rangle}$$

$$h\_tw := DB^{\langle 20 \rangle} \qquad F'''_y := DB^{\langle 21 \rangle}\cdot ksi \qquad X_1 := DB^{\langle 22 \rangle}\cdot ksi$$

$$X_2 := DB^{\langle 23 \rangle}\cdot\frac{10^{-6}}{ksi^2} \qquad I_x := DB^{\langle 24 \rangle}\cdot in^4 \qquad Z_x := DB^{\langle 25 \rangle}\cdot in^3$$

$$S_x := DB^{\langle 26 \rangle}\cdot in^3 \qquad r_x := DB^{\langle 27 \rangle}\cdot in \qquad I_y := DB^{\langle 28 \rangle}\cdot in^4$$

$$Z_y := DB^{\langle 29 \rangle}\cdot in^3 \qquad S_y := DB^{\langle 30 \rangle}\cdot in^3 \qquad r_y := DB^{\langle 31 \rangle}\cdot in$$

$$r_z := DB^{\langle 32 \rangle}\cdot in \qquad J := DB^{\langle 33 \rangle}\cdot in^4 \qquad C_w := DB^{\langle 34 \rangle}\cdot in^6$$

$$W_{no} := DB^{\langle 35 \rangle}\cdot in^2 \qquad S_w := DB^{\langle 36 \rangle}\cdot in^4 \qquad Q_f := DB^{\langle 37 \rangle}\cdot in^3$$

$$Q_w := DB^{\langle 38 \rangle}\cdot in^3 \qquad r_0 := DB^{\langle 39 \rangle}\ in \qquad H := DB^{\langle 40 \rangle}$$

$$index_W(d_{nom}, w_{nom}) := \left| \begin{array}{l} cond \leftarrow \overrightarrow{\left[\left(DB^{\langle 1 \rangle} = 1\right)\cdot\left(d_{nom} = DB^{\langle 2 \rangle}\right)\cdot\left(w_{nom} = DB^{\langle 3 \rangle}\right)\right]} \\ \text{for } i \in 1 .. rows(DB) \\ \quad j \leftarrow i \text{ if } cond_i \\ \text{"Not in file" if } \sum cond = 0 \\ j \text{ otherwise} \end{array} \right.$$

Figure 7 AISC_GetShapes v3

rows (each row corresponds to one shape) are identified by an index number (i). Now the functions BC and BC_{index} can access any tabulated geometric property for a given shape by calling a geometric property name with the matrix subscript i. The final output solution list is then run through a function (shown in Appendix A) which presents the cross sections as text in their standard AISC naming convention (W12 x 45 for example is a W-Shape with a nominal depth of 12 inches weighing 45 pounds per linear foot).

Analyzing the Selected Member

Once the design portion of the sheet has returned the requested number of adequate sections in the given nominal depth, the user then has the option to analyze the member for the given design parameters. This allows the user to explicitly see how the shape satisfies the interaction equation. Additionally, it allows the user to use the Design and Analysis Wizard for analyzing any selected member for the conditions entered in the design portion of the wizard.

The analysis portion of the wizard makes use of Mathcad's Excel component and Excels drop down list control box. Once the user selects the W-shape from the list, the W-shapes index number (i) is returned to Mathcad so it may be used with the AISC_GetShapes v3 file to retrieve that shapes geometric properties. Those properties are then displayed in Mathcad. Then the wizard makes use of the Findb, Findn, and Findm functions used in the design portion. By calling these functions in the variable definitions shown in Figure 8, the member's axial compressive, and both flexural design strengths can be calculated. Finally the interaction value (IA) is calculated using the routine shown in Figure 9. Two example problems are provided in Appendix B. These two problems were taken from the AISC Manual of Steel Construction 3rd Edition and the 4th Edition of Salmon and Johnson.

$$IA := \begin{cases} \frac{P_u}{\phi P_{ncomp}} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) & \text{if } \frac{P_u}{\phi P_{ncomp}} \geq .2 \\ \frac{P_u}{2\ \phi P_{ncomp}} + \left(\frac{M_{ux}}{\phi M_{nx}} + \frac{M_{uy}}{\phi M_{ny}}\right) & \text{otherwise} \end{cases}$$

Figure 9 Interaction Equations in Mathcad

PEDAGOGIC APPLICATIONS

The primary reason we developed the wizard was to aid in the creation of problem sets and tests as well as to check student design problems. It, as well as the functions that it uses, have paid dividends in other areas as well. Since the functions and AISC database are in the Mathcad environment, they have lent themselves to use both on our course websites and in class. The Mathcad environment allows students and instructors alike to change variables and see how the change would effect the final solution. Additionally Mathcad allows graphing of functions to show how design

$$\phi P_{ncomp} := \frac{1}{\text{Findb}\left(F_{cr}\left(K_x, K_y, l_x, l_y, F_y, i\right), i\right)}$$

$$\phi M_{ny} := \frac{8}{9} \cdot \frac{1}{\text{Findn}\left(M_{py}\left(F_y, i\right), M_{nFLB}\left(M_{py}\left(F_y, i\right), M_{ry}\left(F_L\left(F_y, F_r\right), i\right), F_y, F_L\left(F_y, F_r\right), S, i\right)\right)}$$

$$\phi M_{nx} := \frac{8}{9} \cdot \frac{1}{\text{Findm}\left(M_{ncompact}\left(C_b, L_b, L_p\left(F_y, i\right), L_r\left(F_y, F_r, i\right), M_{px}\left(F_y, i\right), M_{rx}\left(F_L\left(F_y, F_r\right), i\right), i\right), M_{nFLB}\left(M_{px}\left(F_y, i\right), M_{rx}\left(F_L\left(F_y, F_r\right), i\right), F_y, F_L\left(F_y, F_r\right), S_{x_i}, i\right)\right)}$$

Figure 8 Variable Definitions for Member Design Strengths strength can change with any independent variable.

Perhaps the most obvious, yet overlooked, benefit of using the wizard is its capability to interact directly with the AISC database. Just by referencing the AISC_GetShapesv_3 file into any Mathcad sheet we have complete access to the AISC shapes database. This allows quick and error-free use during class and for developing out of class assignments. I have saved myself countless hours of looking up geometric properties by using this file. Additionally, we are able to change all member properties in a given problem simply by changing the shape called in the problem. This has been a great resource for both in and out of class assignments and presentations.

I have also used the sheet to solve my semester-long design problem. There are large commercial design packages which will design structures, but for our relatively small structures we focus on an element by element design procedure (with the necessary iteration for moment resisting frames). The wizard is extremely helpful in displaying such solutions. By showing all the element geometric and loading conditions, and by displaying as many possible solutions as the user wants, the wizard can show all cross sections that will satisfy the strength design for member compression, flexure, or even tension limit states. Once the parameters of the problem are input, the worksheet can be saved under a unique name or as a picture and displayed in a solution.

The same features of the spreadsheet make it ideal for spot-checking student capstone designs. These designs are typically unique from group to group, so going through each design to check its accuracy and correctness is difficult and very time consuming. By using the wizard to check elements of an overall design, I can quickly test key members to see if they are adequate for the parameters of any project. This allows me to focus my grading and allows me to provide some great feedback to students on their design.

As an additional benefit, we have used the functions, in combination with Mathcad's graphing capability, to show how different parameters affect design strength. As an example, Figure 10 shows how, using the functions provided, the buckling strength of any selected W-shape can change by altering the grade of steel, effective length factor, length, or axis about which buckling occurs. By using this sheet in class, students can be shown how these parameters can effect compressive strength.

Finally some instructors may wish to give their students access to the functions and wizard in order to help them to speed the design individual members. For advanced design or structural systems classes this may be particularly helpful in showing how members interact in a gravity or lateral system without spending a lot of time designing individual members.

REFERENCES

1. Aminmansour, Abbas. "A New Approach for Design of Steel Beam-Columns." Engineering Journal 37 (2000): 41-72.

2. American Institute of Steel Construction. Manual of Steel Construction – Load and Resistance Factor Design, 3rd Edition, Chicago, IL.

3. Salmon, Charles, and John Johnson. Steel Structures Design and Behavior. 4th ed. New York: Harper Collins, 1996.

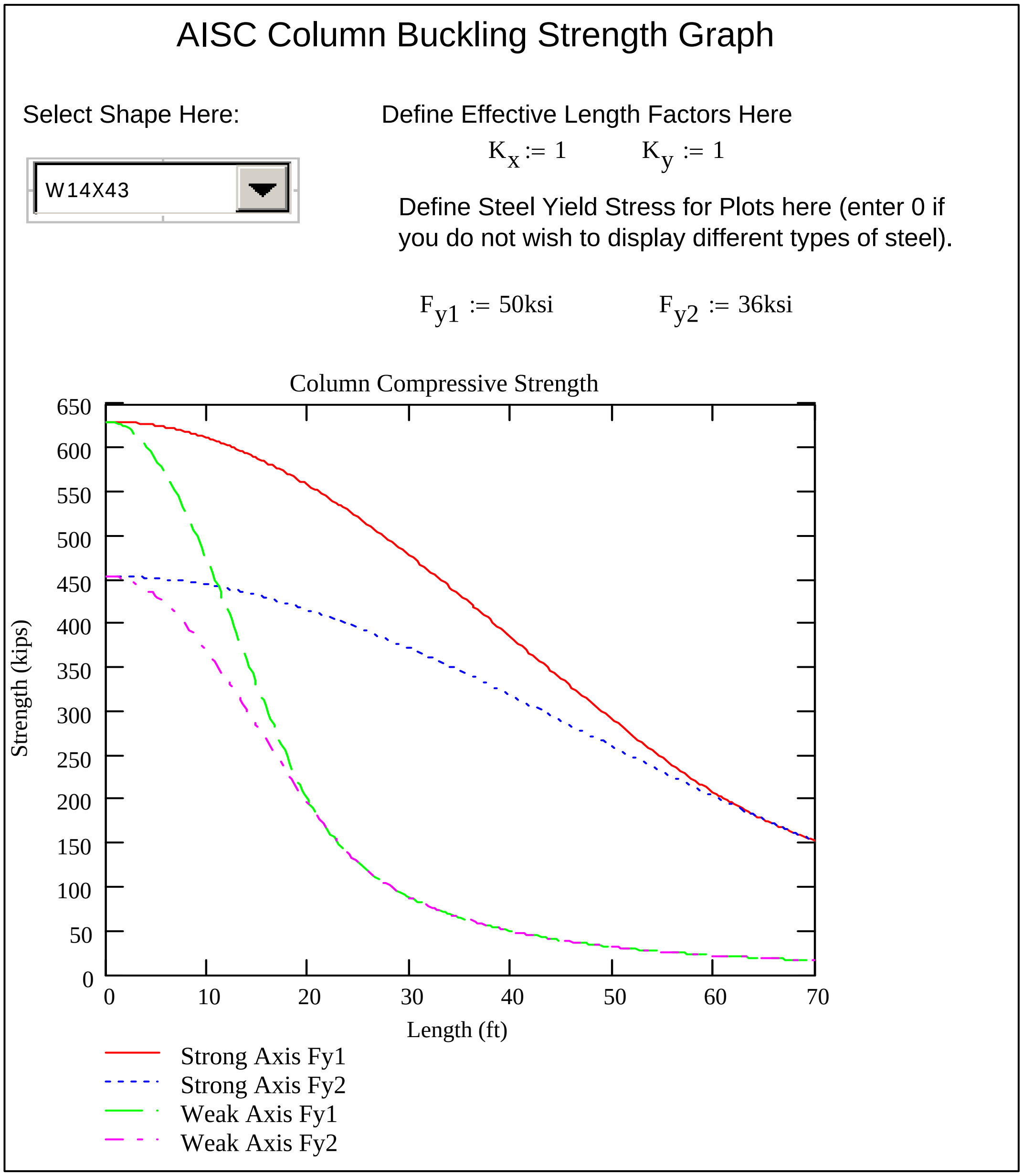

Figure 10 Column Buckling Chart

BIOGRAPHICAL INFORMATION

Major Quadrato is an assistant professor at the United States Military Academy. Craig teaches structural steel design and the civil engineering capstone design project. He is a 1991 graduate of USMA and holds Master of Science degrees in Engineering Management from the University of Missouri and Structural Engineering and Construction Engineering Management from Stanford University. He is a member of ASEE, AISC, and ASCE and serves as a trustee of the Mid-Hudson branch of ASCE.

Major Chris Schirner is currently serving as the Operations Officer for the 84th Engineer Battalion. Chris previously served as an assistant professor at the United States Military Academy and taught structural steel design, statics and dynamics, and soil mechanics and foundation design. He is a 1989 graduate of the United States Military Academy with a B.S. in Civil Engineering and holds a Master's Degree from the University of Maryland in Structural Engineering.

Appendix A
Mathcad Functions

Axial Compressive Design Strength Functions

The following function returns the b value for a W-shape given the critical buckling stress (F_{cr}) in ksi and index number for shape (i) from the AISC shapes database. This function does not include local buckling reductions ($\lambda < \lambda_r$) for pure compression. Below the function is an example of how to use the function by nesting all other possible functions provided in the steel design functions package.

$$\text{Findb}\left(F_{cr}, i\right) := \frac{1}{0.85 \cdot F_{cr} \cdot A_i}$$

$$\text{Findb}\left(F_{cr}\left(K_x, K_y, l_x, l_y, F_y, i\right), i\right)$$

The following function returns the critical stress for flexural buckling of compression members whose elements have width-thickness ratios less than λ_r when given K_x, K_y, l_x, l_y, F_y, and i.

$$F_{cr}\left(K_x, K_y, l_x, l_y, F_y, i\right) := \begin{array}{|l} Kl/r_{max} \leftarrow \max\left(\dfrac{K_x \cdot l_x}{r_{x_i}}, \dfrac{K_y \cdot l_y}{r_{y_i}}\right) \\ \lambda_c \leftarrow \dfrac{Kl/r_{max}}{\pi} \cdot \sqrt{\dfrac{F_y}{E}} \\ F_{cr} \leftarrow \begin{array}{|l} 0.658^{\lambda_c^{\,2}} \cdot F_y \quad \text{if } \lambda_c \le 1.5 \\ \dfrac{0.877}{\lambda_c^{\,2}} \cdot F_y \quad \text{otherwise} \end{array} \end{array}$$

Weak Axis (y-y) Flexural Design Strength Functions

The following function returns the n-value for a W-shape given the fully plastic moment about the weak axis (M_{py}) in kip-ft and index number for shape (i) from the AISC shapes database. Included in the calculation is the reduction for flange local buckling when required. Below the function is an example of how to use the function by nesting all other possible functions provided in the steel design functions package.

$$\text{Findn}\left(M_{py}, M_{nFLB}\right) := \frac{8}{9} \cdot \frac{1}{.9 \cdot \min\left(\begin{pmatrix} M_{py} \\ M_{nFLB} \end{pmatrix}\right)}$$

$$\text{Findn}\left(M_{py}\left(F_y, i\right), M_{nFLB}\left(M_{py}\left(F_y, i\right), M_{ry}\left(F_L\left(F_y, F_r\right), i\right), F_y, F_L\left(F_y, F_r\right), S, i\right)\right)$$

The following function returns the fully plastic moment about the weak axis for compact shapes when given yield stress (F_y) in ksi, and index number for shape (i).

$$M_{py}(F_y, i) := \min\begin{pmatrix} Z_{y_i} \cdot F_y \\ 1.5 \cdot S_{y_i} \cdot F_y \end{pmatrix}$$

The following function returns the nominal bending strength about the axis of consideration for the limit state of FLB for a doubly symmetric shape or channel given M_p in kip-ft, M_r in kip-ft, elastic section modulus (S) in in^3 and index number for shape (i). Note: put in M_p, M_r, and S about the axis of consideration (either x-x or y-y).

$$M_{nFLB}(M_p, M_r, F_y, F_L, S, i) := \left[\begin{array}{ll} M_p & \text{if } bf\_2tf_i \le .38 \cdot \sqrt{\frac{E}{F_y}} \\ M_p - (M_p - M_r) \cdot \dfrac{\left(bf\_2tf_i - .38 \cdot \sqrt{\frac{E}{F_y}}\right)}{\left(.83 \cdot \sqrt{\frac{E}{F_L}} - .38 \cdot \sqrt{\frac{E}{F_y}}\right)} & \text{if } \left(.38 \cdot \sqrt{\frac{E}{F_y}} < bf\_2tf_i \le .83 \cdot \sqrt{\frac{E}{F_L}}\right) \\ \min\left[\begin{bmatrix} S \cdot \dfrac{.69 \cdot E}{(bf\_2tf_i)^2} \\ M_p \end{bmatrix}\right] & \text{otherwise} \end{array}\right]$$

The following function returns the limiting buckling moment about the weak axis when $\lambda <= \lambda_r$ when given F_L in ksi, and index number for shape (i).

$$M_{ry}(F_L, i) := S_{y_i} \cdot F_L$$

The following function returns the residual stress affected yeild strength for a cross-section made from a uniform grade of steel when given yield stress (F_y) in ksi, and residual stress (F_r) in ksi.

$$F_L(F_y, F_r) := (F_y - F_r)$$

Strong Axis (x-x) Flexural Design Strength Functions

The following function returns the m-value for a W-shape given the nominal moment based on the limit state of lateral torsional buckling ($M_{ncompact}$) and flange local buckling (M_{nFLB}) about the strong axis (M_{py}) in kip-ft and index number for shape (i) from the AISC shapes database. Below the function is an example of how to use the function by nesting all other possible functions provided in the steel design functions package.

$$\text{Findm}(M_{ncompact}, M_{nFLB}) := \frac{8}{9 \cdot .9 \cdot \min\left(\begin{pmatrix} M_{ncompact} \\ M_{nFLB} \end{pmatrix}\right)}$$

$$\text{Findm}\left(M_{ncompact}\left(C_b, L_b, L_p(F_y, i), L_r(F_y, F_r, i), M_{px}(F_y, i), M_{rx}(F_L(F_y, F_r), i), i\right), M_{nFLB}\left(M_{px}(F_y, i), M_{rx}(F_L(F_y, F_r), i), F_y, F_L(F_y, F_r), S_{x_i}, i\right)\right)$$

The following function returns the nominal bending strength based on the limit state of lateral torsional buckling about the strong axis for a compact doubly symmetric shape or channel given bending coefficient (C_b), unbraced length (L_b) in ft, L_p in ft, L_r in feet, M_p in kip-ft, M_r in kip-ft, and index number for shape (i).

$$M_{ncompact}(C_b, L_b, L_p, L_r, M_{px}, M_{rx}, i) := \begin{cases} M_{px} & \text{if } L_b \le L_p \\ \min\left[\begin{pmatrix} C_b \cdot \left[M_{px} - (M_{px} - M_{rx}) \cdot \dfrac{(L_b - L_p)}{(L_r - L_p)}\right] \\ M_{px} \end{pmatrix}\right] & \text{if } (L_p < L_b \le L_r) \\ \min\left[\begin{pmatrix} C_b \cdot \dfrac{S_{x_i} \cdot X_{1_i} \cdot \sqrt{2}}{\dfrac{L_b}{r_{y_i}}} \cdot \sqrt{1 + \dfrac{\left(X_{1_i}\right)^2 \cdot X_{2_i}}{2 \cdot \left(\dfrac{L_b}{r_{y_i}}\right)^2}} \\ M_{px} \end{pmatrix}\right] & \text{otherwise} \end{cases}$$

The following function returns the limiting laterally unbraced length for full plastic bending capacity (L_p) assuming C.b = 1.0 when given yield stress (F_y) in ksi, and index number for shape (i).

$$L_p(F_y, i) := 1.76 \, r_{y_i} \cdot \sqrt{\frac{E}{F_y}}$$

The following function returns the limiting laterally unbraced length for inelastic lateral-torsional buckling (L_r) when given yield stress (F_y) in ksi, residual stress (F_r) in ksi, and index number for shape (i).

$$L_r(F_y, F_r, i) := \frac{r_{y_i} \cdot X_{1_i}}{F_y - F_r} \cdot \sqrt{1 + \sqrt{1 + X_{2_i} \cdot (F_y - F_r)^2}}$$

The following function returns the fully plastic moment about the strong axis for compact shapes when given yield stress (F_y) in ksi, and index number for shape (i).

$$M_{px}(F_y, i) := \min\begin{pmatrix} Z_{x_i} \cdot F_y \\ 1.5 \, S_{x_i} \cdot F_y \end{pmatrix}$$

The following function returns the limiting buckling moment about the strong axis when $\lambda <= \lambda_r$ and $C_b = 1.0$ when given F_L in ksi, and index number for shape (i).

$$M_{rx}(F_L, i) := S_{x_i} \cdot F_L$$

<u>Other Functions</u>

This function produces a vector of members names when given a table of members taken from the AISC shapes database (currently it only accepts W and LL members).

```
Members(Table) := | for  i ∈ 1 .. rows(Table)
                  |   | Members_i ← concat["W" , num2str[(Table<2>)_i], " x " , num2str[(Table<3>)_i]]  if (Table<1>)_i = 1
                  |   | Members_i ← concat[concat["LL" , num2str[(Table<5>)_i], " x " , num2str[(Table<8>)_i]], " x " , num2str[(Table<12>)_i]]  if (Table<1>)_i = 11
                  | Members
```

Appendix B
Example Problems

<u>Example 6.4 from AISC Manual of Steel Construction 3rd Edition</u>.

Select the lightest ASTM A992 W14 (F_y = 50 ksi, F_u = 65 ksi) that is adequate for the following combination of axial compression and flexure:

P_u = 400kips M_{ux} = 250 kip-ft
M_{uy} = 80 kip-ft Assume $KL_x = KL_y = L_b$ = 14 ft

(Answer W14 x 99 with Interaction Value of 0.951).

W-Shape Design and Analysis Wizard

Enter the following information:

Number of Sections to Report: $NumSect := 1$ Nominal Depth to use: $d_n := 14$

Loads	End Conditions	Lengths	Loading Condition	Steel Values
$M_{ux} := 250ft \cdot kips$	$K_y := 1.0$	$L_b := 14ft$	$C_b := 1$	$F_y := 50ksi$
$M_{uy} := 80ft \cdot kips$	$K_x := 1$	$l_x := 14ft$		$F_r := 10ksi$
$P_u := 400kips$ Compression Only		$l_y := 14ft$		

The member or members that satisfy the above conditions are: Member = ("W14 x 99")

W14X99

To perform an analysis of this member select it member from the drop-down list to the right by double clicking the list to activate it then select member from the drop down list. Then click in the Mathcad worksheet area to update this sheet.

Common Geometric Properties of selected shape are listed below

A =	29.1	in^2	h_tw =	3190		r_x =	5.23	in
d =	14.2	in	Fy''' =	0.00122	ksi	Z_x =	77.9	in3
t_w =	0.485	in	X_1 =	1110	ksi	I_y =	95.8	in^4
b_f =	14.6	in	X_2 =	157	$1/ksi^2$	S_y =	19.2	in^3
t_f =	0.78	in	I_x =	6.17	in^4	r_y =	2.48	in
bf_2tf =	9.34		S_x =	173	in^3	Z_y =	29.1	in^3

Axial Compressive Design Strength

$\phi P_{ncomp} = 1065 kips$

Note ϕPn above does not include local buckling check or slenderness considerations.

Axial Tensile Design Strength (for yeild in gross area)

$\phi P_{nyld} = 1310 kips$

Flexural Design Strength

$\phi M_{nx} = 643 kips \cdot ft$

$\phi M_{ny} = 308 kips \cdot ft$

Interaction Value (assuming axial compression)

$IA = 0.952$

Note this interaction value does not include possible local buckling of web under combined flexural and axial compression

Example 12.13.3 from Salmon and Johnson's text, Steel Structures Design and Behavior, 4th Edition (also Example 1 from Aminmansour).

Given: $P_u = 179$ kips, $M_{ux} = 53.3$ kip – ft, $M_{uy} = 0$ kip – ft, A36 Steel
$KL_x = 16$ ft, $KL_y = Lb = 8$ ft, and Let $C_b = 1.0$

(Answer: Select W10x39 with interaction value (IA) of 0.95)

W-Shape Design and Analysis Wizard

Enter the following information:

Number of Sections to Report: NumSect := 1

Nominal Depth to use: $d_n := 10$

Loads		End Conditions	Lengths	Loading Condition	Steel Values
$M_{ux} := 53.3 ft \cdot kips$		$K_y := 1.0$	$L_b := 8ft$	$C_b := 1$	$F_y := 36ksi$
$M_{uy} := 0ft \cdot kips$		$K_x := 1$	$l_x := 16ft$		$F_r := 10ksi$
$P_u := 179kips$	Compression Only		$l_y := 8ft$		

The member or members that satisfy the above conditions are: Member = ("W10 x 39")

W10X39

To perform an analysis of this member select it member from the drop-down list to the right by double clicking the list to activate it then select member from the drop down list. Then click in the Mathcad worksheet area to update this sheet.

Common Geometric Properties of selected shape are listed below

A =	11.5	in^2	h_tw =	3190		r_x =	5.23	in
d =	9.92	in	Fy''' =	0.00131	ksi	Z_x =	77.9	in3
t_w =	0.315	in	X_1 =	209	ksi	I_y =	95.8	in^4
b_f =	7.99	in	X_2 =	42.1	$1/ksi^2$	S_y =	19.2	in^3
t_f =	0.53	in	I_x =	4.27	in^4	r_y =	2.48	in
bf_2tf =	7.53		S_x =	46.8	in^3	Z_y =	29.1	in^3

Axial Compressive Design Strength

$\phi P_{ncomp} = 311kips$

Note ϕPn above does not include local buckling check or slenderness considerations.

Axial Tensile Design Strength (for yeild in gross area)

$\phi P_{nyld} = 373kips$

Flexural Design Strength

$\phi M_{nx} = 126kips \cdot ft$

$\phi M_{ny} = 46kips \cdot ft$

Interaction Value (assuming axial compression)

IA = 0.951

Note this interaction value does not include possible local buckling of web under combined flexural and axial compression