Fin Calculations in Mathcad

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Introduction

When heat is being transferred to or from a gas, it is often cost effective to add fins to the heat transfer surface. These fins increase the surface area and thus increase the rate of heat transfer. The customary method of calculating the heat transfer from fins is to begin with the assumption that the entire fin is at the temperature of the base where the fin is attached to the original surface. In this ideal case, the rate of heat transfer is easily calculated. For the actual case, the ideal rate of heat transfer is multiplied by an efficiency factor.

Fin efficiency factors are derived from the analytical solution of the differential equation that governs the rate of heat transfer. Because the resulting equations are quite complicated, the typical heat transfer textbook presents the results in the form of fin efficiency charts. There is some loss of precision in using the charts rather than the equations, but this is unimportant in view of the uncertainty in the convection coefficient.

When automating fin calculations with a tool such as Mathcad, it is inconvenient to have to refer to the charts for fin efficiency values. This article presents a Mathcad worksheet that calculates fin heat transfer rates using efficiencies obtained from the governing equations. These equations are taken from the work of Gardner[1].

Governing Equations

For circumferential fins on a tube of circular cross-section, the exact solution is based on the parameters U_b and U_e that are defined as follows:

$$U_b = \frac{L\sqrt{2h/kt}}{(r_o/r_i)-1}$$

and

$$U_e = U_b \frac{r_o}{r_i}$$

where L is the radial fin length, h is the convection coefficient, k is the fin thermal conductivity, t is the fin thickness, r_o is the radius of the fin tip, and r_i is the radius of the fin root. The parameter β is defined as follows:

$$\beta = \frac{I_1(U_e)}{K_1(U_e)}$$

where I_1 and K_1 are Bessel functions. Finally, the fin efficiency η is calculated as follows:

$$\eta = \frac{2}{U_b\left[1-\left(\frac{U_e}{U_b}\right)^2\right]} \frac{I_1(U_b)-\beta K_1(U_b)}{I_0(U_b)+\beta K_0(U_b)}$$

where I_0 and K_0 are also Bessel functions. In addition the fin effectiveness ε can be shown to be

$$\varepsilon = \frac{\eta\left(r_o^2 - r_i^2\right)}{r_i t}.$$

Particularly because of the Bessel functions involved in these equations, it is not convenient to use them if calculations are being done on a calculator. However, they are easily implemented in Mathcad, because Bessel functions are built in.

Mathcad Worksheet

The Mathcad worksheets shown in Figures 1 through 3 are extensively annotated to make them self-explanatory. Calculated values do not appear in the figures, because the cut and paste process does not transfer them.

Figure 1 shows the portion of the Mathcad worksheet that defines functions for fin efficiency and effectiveness. This portion of the worksheet is made available for students to use in solving their own problems. This worksheet and many others are available to the general public at the following URL.

http://www.webb-insistute.edu/ewiggins/ME5/where_me5_files_are.htm.

Figure 2 shows the beginning of an example problem. This problem is based on problem 3.100 in Incropera and De Witt[2] In this figure, the geometry and heat transfer input parameters are defined, and the Mathcad functions that were defined in Figure 1 are invoked to calculate the efficiency and effectiveness of the fins.

Figure 3 shows the conclusion of the example problem that begins in Figure 2. The actual heat transfer rate from one fin is about 21 watts, and the total heat transfer rate from one meter of finned pipe is about 4.5 kilowatts.

The portion of the worksheet shown in Figure 1 can be followed by any particular problem involving circumferential fins. One need only supply values for the arguments of the efficiency and effectiveness functions, and Mathcad will do the rest.

Conclusions

The first part of this Mathcad worksheet is an electronic replacement for the fin efficiency charts found in most heat transfer textbooks. I place a copy of it on the course Website and encourage students to download it and use it instead of the charts. They simply append their model for a particular problem to the first part of the worksheet, and Mathcad does the calculations. Because the calculation of efficiency is built in, they can quickly and easily explore the effect of varying fin parameters such as length and thermal conductivity. This facilitates a deeper understanding of fin design.

References

1. Gardner, Karl A., “Efficiency of Extended Surface,” Transactions of the ASME, Nov. 1945, p. 621.

2. Incropera, F. P. and D. P. De Witt, Introduction to Heat Transfer, 2nd edition, John Wiley & Sons, 1990, p. 169.

Biographical Information

Edwin G. Wiggins holds BS, MS, and Ph.D. degrees in chemical, nuclear, and mechanical engineering respectively from Purdue University. He is the Mandell and Lester Rosenblatt Professor of Marine Engineering at Webb Institute in Glen Cove, NY. Ed is a past chairman of the New York Metropolitan Section of the Society of Naval Architects and Marine Engineers (SNAME) and a past regional vice president of SNAME. A Centennial Medallion and a Distinguished Service Award recognize his service to SNAME. As a representative of SNAME, Ed Wiggins serves on the Engineering Accreditation Commission of the Accreditation Board for Engineering and Technology.

Figure 1
Definition of Fin Efficiency and Effectiveness Functions

Efficiency and effectiveness of a fin of rectangular profile on a circular tube

$C := K$ This definition makes it possible to work in Celsius rather than Kelvin

Here are the equations for fin efficiency and effectiveness

$$U_b(L_f, h, k_f, t_f, \mathrm{radratio}) := \frac{L_f \cdot \sqrt{\dfrac{2 \cdot h}{k_f \cdot t_f}}}{\mathrm{radratio} - 1}$$

$$U_e(L_f, h, k_f, t_f, \mathrm{radratio}) := U_b(L_f, h, k_f, t_f, \mathrm{radratio}) \cdot \mathrm{radratio}$$

$$\beta(L_f, h, k_f, t_f, \mathrm{radratio}) := \frac{I1(U_e(L_f, h, k_f, t_f, \mathrm{radratio}))}{K1(U_e(L_f, h, k_f, t_f, \mathrm{radratio}))}$$

Fin efficiency

$$\eta_f(L_f, h, k_f, t_f, \mathrm{radratio}) := \frac{2}{U_b(L_f, h, k_f, t_f, \mathrm{radratio}) \cdot \left[1 - \left(\dfrac{U_e(L_f, h, k_f, t_f, \mathrm{radratio})}{U_b(L_f, h, k_f, t_f, \mathrm{radratio})}\right)^2\right]} \cdot \frac{I1(U_b(L_f, h, k_f, t_f, \mathrm{radratio})) - \beta(L_f, h, k_f, t_f, \mathrm{radratio}) \cdot K1(U_b(L_f, h, k_f, t_f, \mathrm{radratio}))}{I0(U_b(L_f, h, k_f, t_f, \mathrm{radratio})) + \beta(L_f, h, k_f, t_f, \mathrm{radratio}) \cdot K0(U_b(L_f, h, k_f, t_f, \mathrm{radratio}))}$$

Fin effectiveness

$$\varepsilon_f(L_f, h, k_f, t_f, \mathrm{radratio}, r_o, r_i) := \frac{\eta_f(L_f, h, k_f, t_f, \mathrm{radratio}) \cdot \left(r_o^2 - r_i^2\right)}{r_i \cdot t_f}$$

Figure 2
Fin Example Problem – Part 1

Given: A circumferential aluminum fin fin of rectangular profile is attached to a circular tube of outside diameter 25 mm and surface temperature 250 C. The fin is 1 mm thick and 10 mm long. The surrounding fluid is at 25 C with convection coefficient equal to 25 W/(m^2 C).

Find: The heat loss per fin. If 200 fins are spaced at 5-mm increments along the tube length, what is the heat loss per meter of finned tube?

Solution:

Fin geometry

$r_i := 12.5 \cdot mm$ Inside fin radius

$L_f := 15 \cdot mm$ Fin length

$r_o := r_i + L_f$ $r_o = 27.5mm$ Outside fin radius

$\text{radratio} := \frac{r_o}{r_i}$ $\text{radratio} = \blacksquare$

$t_f := 1 \cdot mm$ Fin thickness

Heat transfer parameters

$T_0 := 250 \cdot C$ Wall temperature and fin base temperature

$T_f := 25 \cdot C$ Fluid temperature

$h := 25 \cdot \frac{W}{m^2 \cdot C}$ Convection coefficient

$k_f := 240 \cdot \frac{W}{m \cdot C}$ Thermal conductivity of fin

Fin efficiency and effectiveness

$\eta := \eta_f\left(L_f, h, k_f, t_f, \text{radratio}\right)$ $\eta = \blacksquare$

$\varepsilon := \varepsilon_f\left(L_f, h, k_f, t_f, \text{radratio}, r_o, r_i\right)$ $\varepsilon = \blacksquare$

Figure 3
Fin Example Problem – Part 2

Heat transfer calculations

$A_f := 2 \cdot \pi \cdot \left(r_o^2 - r_i^2\right)$ $A_f = ▮$ Fin surface area - top and bottom

$q_{max} := h \cdot A_f \cdot \left(T_0 - T_f\right)$ $q_{max} = ▮$ Heat transfer if entire fin at base temperature

$q_f := \eta \cdot q_{max}$ $\boxed{q_f = ▮}$ Actual heat transfer from one fin

If 200 1-mm fins are spaced at 5 mm intervals, there is 0.8 meters of bare tube surface. We need to calculate the heat transfer from that and combine it with the heat transferred from the fins.

$q_w := h \cdot 2 \cdot \pi \cdot r_i \cdot 0.8 \cdot m \cdot \left(T_0 - T_f\right)$ $q_w = ▮$ Heat transfer from bare wall

$q_{200f} := 200 \cdot q_f$ $q_{200f} = ▮$ Heat transfer from all fins

$q := q_w + q_{200f}$ $\boxed{q = ▮\ kW}$ Total heat transfer per meter of pipe

If there were no fins at all

$q_{bare} := h \cdot 2 \cdot \pi \cdot r_i \cdot 1 \cdot m \cdot \left(T_0 - T_f\right)$ $q_{bare} = ▮$